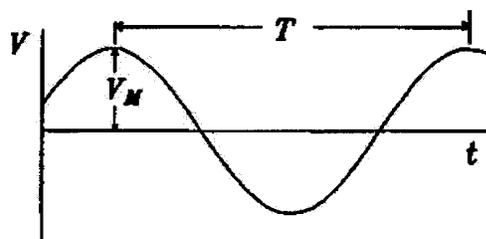
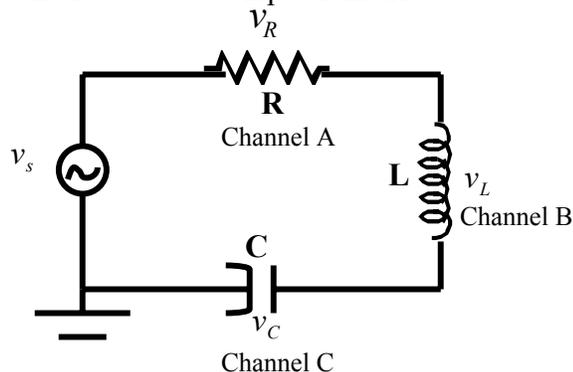


PHASE AND IMPEDANCE IN AN RLC CIRCUIT

The purpose of this experiment is to introduce you to some of the phenomena and terminology of AC circuits involving resistance, inductance, and capacitance. In making measurements you will use the digital storage oscilloscope and signal generator capabilities of the *Pasco 750* computer interface.



THE EXPERIMENT

By means of banana-plug-tipped wires connect the series RLC circuit shown above. For the source, S , you will use the signal generator on the interface (marked “OUTPUT”). Use $R = 1\text{k}\Omega$ and $C = 1\mu\text{f}$. For the inductance, L , use the large copper coil on the clear plastic spool.

The three analog channels of the interface should have voltage probes, each consisting of red and black wires tipped by banana plugs. Connect these probes so that they will measure the voltages across R (connected to channel A), L (connected to channel B), and C (connected to channel C). To make your voltages have the correct sign, you need to maintain polarity by connecting each black lead to the “side” of the circuit element “nearest” to ground.

THE OSCILLOSCOPE/COMPUTER SETUP

When your connections are complete and the computer is on, use the following computer operations to activate and adjust the oscilloscope:

- Double click on the “Data Studio” icon.
- At the main menu choose “Create Experiment”.
- In the “Sensors” window scroll down and double click “Voltage Sensor” three times. This should place voltage sensors in channels A, B, and C on the picture of your interface.
- Under “Signal Output” double click on “Output”.
- Set the amplitude to 1 V and set the frequency to 100 Hz.
- In the “Displays” window, in the bottom left corner, double click on “Scope”. Choose “Voltage, ChA”. This should give you a scope with channel A voltage on the y-axis.
- In the top left “Data” window click and drag “Voltage, ChB” and “Voltage, ChC” into your scope. This should give you a scope that will display all three voltages on the y-axis and time on the x-axis.
- To begin, simply click the [Start] button.

At this point you should carefully determine and write down the channel and color of trace that corresponds to each of the circuit elements. Later you will need to know at a glance

which voltage you are observing. You should also change the scaling on your scope so that you can clearly distinguish the waveforms (a good idea for this lab would be to set all of the “V/div” to 0.5 V/div and set the “ms/div” to 2 ms/div). To stabilize your scope click on the triggering icon () at the top of your graph.

You should now have a relatively stable display with each new trace being triggered by a rise in the A signal. Click and drag your scope down a little so that the “Signal Generator” window is exposed. Vary the amplitude and frequency of the signal generator and note the change in the display. Notice what device dominates at high frequencies and what dominates at low frequencies. Why do you think this happens (explain in your report)? Now set the signal generator back to 1 V and 100 Hz and hit stop.

PHASE MEASUREMENTS

Each of the voltages in your circuit (v_R , v_L , v_C , and v_s , the source voltage) has the form

$$v(t) = V_{\max} \sin(\omega t + \phi),$$

where V_{\max} is the amplitude (the maximum value, the zero-to-peak voltage). The *phase* is $\omega t + \phi$, where ω is the angular frequency in radians per second. $\omega = 2\pi f$, where f is the frequency in Hertz. Note that time needed for one cycle or period ($T = 1/f = 2\pi/\omega$) corresponds to $2\pi = 360^\circ$. On the oscilloscope this is the time from one peak of a trace to the next peak on that same trace. Because ϕ is different from one circuit element to the next, their voltages are not in phase, i.e. not in step; they do not reach peak values at the same time. For example, a difference in phase of $\pi = 180^\circ$ would correspond to a time difference of $T/2$; one signal would be half a cycle ahead of the other; one would have a peak when the other has a valley.

On your scope click the smart cursor button () at the top left corner. Now drag your mouse to the where the dashed lines cross and click and drag on this area. Use this tracking ability to answer the following questions.

- What is the period, T ? What is the frequency, $f = 1/T$? Is this value consistent with your setting on the signal generator?
- What is the time delay between a peak of V_R and the closest peak of V_C ? What about V_L and V_R ? V_L and V_C ?
- What phase difference, in radians and degrees, does each of these time delays imply?
- Given that the current is the same for all circuit elements and that for the resistor $V = IR$, we have:

$$I(t) = v_R(t) / R.$$

This means that the current is in phase with the voltage across the resistor. What is the phase difference between current and the two other voltages that you are observing? Express your answer in the form “V leads I by ___°” if the peaks of V precede the peaks of I, or “V lags I by ___°” if the peaks of V follow the peaks of I.

IMPEDANCE

The definition of resistance is $R = V_R/I$. This applies to either instantaneous values or maximum values of current and voltage. Therefore,

$$R = V_{RM}/I_M \quad (1)$$

where I_M is the peak current and V_{RM} is the peak voltage across the resistor. *Reactance* and *Impedance* are defined by analogy with this equation. Inductive and capacitive reactance is defined by

$$X_L = V_{LM}/I_M, \quad \text{and} \quad X_C = V_{CM}/I_M. \quad (2)$$

Reactance can be related to capacitance by

$$X_C = 1/(2\pi fC). \quad (3)$$

Similarly, inductive reactance is

$$X_L = 2\pi fL \quad (4)$$

where L is the inductance measured in henries (H).

Measure the peak values of your three voltages. Just in case the “zeroing” of the traces wasn’t complete, measure peak-to-peak (i.e. the difference between the positive and negative peak) voltages and take half. With these values use equation (1) to obtain I_M . Then use (2) to obtain the reactances. As a check, use equation (3) to obtain C . Does this value agree with the capacitance you connected into the circuit? Use equation (4) to obtain L .

The total impedance connected to the source (analogous to the total resistance in a DC circuit) is

$$Z = V_{SM}/I_M. \quad (5)$$

For a series RLC circuit, when the phase differences are properly accounted for, it can be shown that

$$Z = [R^2 + (X_L - X_C)^2]^{1/2} \quad (6)$$

Use R and your values of X_L and X_C in equation (6) to compute Z and use equation (5) to compute the peak source voltage, V_{SM} . In the “Data” window in the top left corner click and drag the “Output Voltage” into your scope. Click [Start] for a few seconds then [Stop]. Determine the peak value of this source voltage. How does this peak value compare with the value you just calculated?

PHASE ANGLE for RLC

A final quantity that is often specified for an RLC circuit is referred to as the phase angle (ϕ). It is not to be confused with the “phase” or “phase shift” of a specific circuit element. Phase angle refers to the whole RLC circuit; it is defined by the equation

$$\tan(\phi) = \frac{X_L - X_C}{R} \quad (7)$$

A positive phase angle corresponds to a circuit that is more inductive in nature and in which the current lags behind the voltage, while a negative phase angle corresponds to a circuit which is more capacitive in nature and in which the current leads the voltage. A popular way to remember this is the saying “ELI the ICE man” which shows the phase relationship between voltage (E) and current (I) for either an inductor (L) or a capacitor (C). *Determine the phase angle for your circuit using the values of R , X_L and X_C .* Now measure this phase angle directly by comparing the relative positions of the output voltage (v_s) trace to the trace for the voltage across the resistor

(v_R). *Compare the two values for the phase angle and explain any difference. Specifically, does the sign of the measured phase angle agree with the expected sign (i.e., positive or negative)?*

Now remove the output voltage trace from your scope by clicking your mouse on where it says “Output Voltage” (bottom right of your scope) and hit backspace (then OK). Then reactivate the oscilloscope to receive new signals (hit [Start]). Observe what happens to your voltages when you place a large piece of iron inside the inductor. *What happens to each of three signals? What does this tell you about the inductance of the coil when the iron core is added? Estimate the new value for the inductance.* Remove the iron before going on.

RESONANCE

According to equation (6), the impedance should be a minimum when $X_L = X_C$. For a fixed V_{SM} this should mean maximum current (resonance). Making use of the fact that the two reactances are frequency dependent, you could vary the frequency to find the value that gives maximum current (as evidenced by the maximum case for a voltage across a resistor, $V_R = IR$). This would be the frequency which makes $V_{LM} = V_{CM}$ (or $v_L(t) = -v_C(t)$). However, specific resonance frequency can be difficult to find by trial and error. So, using this $X_L = X_C$ idea for resonance, set equation (3) equal to (4) and solve for this frequency. Now go to your signal generator window at type in this frequency. Hit [Start] and see if, indeed, your capacitor and inductor have the same amplitude and if $v_L(t) = -v_C(t)$ (this will appear as two waves that are 180° out of phase); hit [Stop]. Were you correct? If you switch the black and red leads on your inductor (channel B) then $v_L(t)$ should appear the same as $v_C(t)$. Activate your scope and check this. Do they match (do they now appear to be in phase and have the same amplitude)?