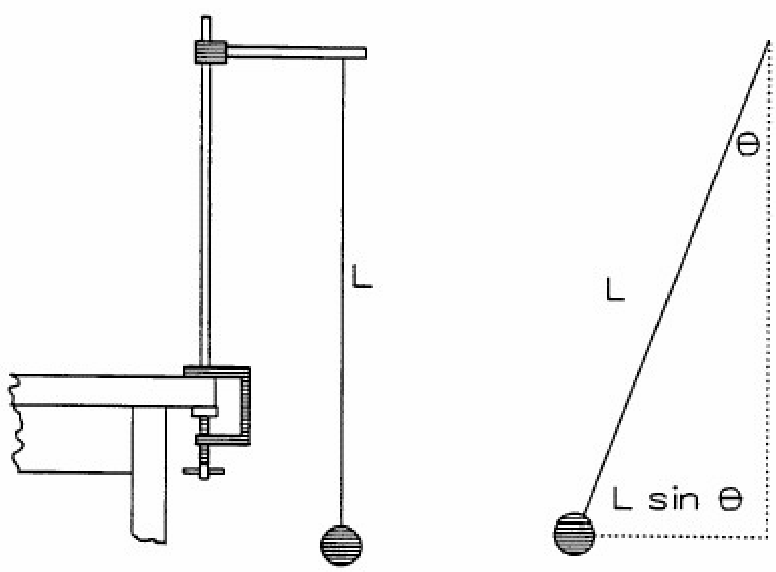
THE SCIENTIFIC METHOD

The purpose of this exercise is to apply a systematic (scientific) approach to a physical problem. A simple pendulum will be used in this particular study. The problem is experimentally to determine whether and how various factors influence the pendulum's period, the time for one complete back-and-forth oscillation. Variables that may or may not affect the period include the mass of the pendulum bob, the length of the string, the amplitude of the angle of swing, and the acceleration due to gravity. The last factor is one over which you have no control. In this study each variable (which you can control) should be isolated and studied individually. Your goal is to write a mathematical expression for the period, T, in terms of the variables that affect it.

The Experiment

(1) First, study the effect of mass. Choose a mass and suspend it by a light string of known length, say 100 cm. The length of the pendulum should be measured from the point of suspension to the center of the mass. Displace the mass to the side until the string makes some angle, say 5º, with the vertical. After releasing the bob use a stop-clock or stopwatch to measure the time for 20 complete vibrations; compute the period, the time for one vibration. (In making your time measurement, start and stop the watch at the midpoint in the swing, i.e. the lowest point. Why is this best? And be sure to count "zero" not "one" when you start the watch!) If you release the mass carefully, you should be able to get 20 oscillations without the mass hitting the table. If not, use fewer oscillations, but not less than 10. Repeat the above procedure for the five different masses. Be very careful to keep the length, maximum angle, and other variables the same for all masses so that you are investigating only the effect of mass.



(2) Second, study the effect of length. Keeping the mass and angle of swing constant, measure the time for 20 oscillations for various lengths such as 10 cm, 20 cm . . . 180 cm, 200cm up to as long as you can get conveniently. You have to use a wide range of lengths to get any useful data out of

this variable, so take several measurements (≥10) across the entire range from 10 to 200 cm. If possible, use some length(s) greater than 200 cm.

(3) Third, study the effect of maximum angle of swing, i.e. the angle at which the pendulum is released. Keeping length and mass constant, measure the time for 20 vibrations for maximum angles from small values up to 90º (use: 5°, 15°, 25°, 35°, 45°, 55°, 65°, & 75°). You should do a couple of runs for the 5° and 15º angles. You may use a protractor in determining the angle or you may find it more precise to use the angle's sine, which is the ratio of the lateral displacement to the pendulum length.

(4) Investigate any other variables you can control.

Determining the Mathematical Relationship

Is the period a function of mass? Of length? Of maximum angle? What is the mathematical relationship in each case? Does any variable have no influence in some range but some influence in another range?

One technique for seeking a mathematical relationship between two variables involves graphing one variable against various powers (or other functions) of the other. For example, if y is directly proportional to $x = x^1$, the graph of y *vs* x will be a straight line with equation of the form

$$\mathbf{y = ax + b,}$$

where a and b are constants; a is the slope (usually represented by "m" when mass is not in consideration) and b is the y-intercept. On the other hand, if y is directly proportional to x^2, the plot of y *vs* x is a parabola but the plot of y *vs* x^2 is a straight line with equation of the form

$$\mathbf{y = ax^2 + b.}$$

Select from your data a variable that influences the period. Plot period *vs.* this variable with other variables held constant. If the plot is not a straight line, try a plot of period vs the square, square root, or other power of the variable. You will know you have found the right power when the graph is a straight line. (With a little practice you may be able to guess the relationship by considering how period is changed when the variable is changed by a factor of 4, 4^2 or $4^{½}$.)

Once you've found which is the linear relationship make sure to state what your constant of proportionality was (your slope). Also, be sure to show all graphs and tables, and circle the graph that you think was the linear relationship.

Using these results try to write a mathematical expression for the period T in terms of mass m, length L, and maximum angle θ. Examples of the kinds of expression you are seeking (the right one is not given) include:

$$T = \frac{am\theta}{L^2}, \qquad T = \frac{bm}{\sqrt{L}}, \qquad T = c\sqrt{\theta},$$

where a, b, and c are constants. Constants can be evaluated by techniques used in analytical geometry. For example, you should be able to get values from the slopes of your straight-line graphs.