Week 9

Micro teaching : 1. Vector Potential

2. $\vec{\nabla}$ in Cartesian

Cylindrical

Spherical coordinates

3. HW 9

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1. Vector potential

We defined $-\nabla\Phi = E$; Similarly we can define $\vec{\nabla}\times\vec{A} = \vec{B}$:

In Electrostatics : $\vec{\nabla}\times\vec{E} = 0 \Rightarrow \vec{\nabla}\times(\vec{\nabla}\Phi) = 0$

In Electrodynamics : $\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

Electrostatics	Electrodynamics
$\nabla\times(\nabla\Phi) = 0$ (Identity)	$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$
$\nabla\Phi = -E \Leftarrow \vec{\nabla}\times\vec{E} = 0$	$\vec{\nabla}\times\vec{A} = \vec{B} \Leftarrow \vec{\nabla}\cdot\vec{B} = 0$

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2. A. $\vec{\nabla}$ in Cartesian coordinates $\{x, y, z\}$

$$\vec{\nabla}\Phi = \langle \partial_x\Phi, \partial_y\Phi, \partial_z\Phi \rangle \qquad \partial_i = \frac{\partial}{\partial i}$$

$$\vec{\nabla}\cdot\vec{A} = \partial_x A_x + \partial_y A_y + \partial_z A_z$$

$$\vec{\nabla}\times\vec{A} = \langle \partial_y A_z - \partial_z A_y, \partial_z A_x - \partial_x A_z, \partial_x A_y - \partial_y A_x \rangle$$

B. $\vec{\nabla}$ in cylindrical coordinates $\{\rho, \phi, z\}$

$$\vec{\nabla} f = \left\langle \partial_\rho f \,,\ \frac{1}{\rho}\cdot \partial_\phi f \,,\ \partial_z f \right\rangle$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\,\partial_\rho(\rho A_\rho) + \frac{1}{\rho}\cdot\partial_\phi A_\phi + \partial_z A_z$$

$$\vec{\nabla}\times\vec{A} = \left\langle \frac{1}{\rho}\partial_\phi A_z - \partial_z A_\phi \,,\ \partial_z A_\rho - \partial_\rho A_z \,,\ \frac{1}{\rho}\left(\partial_\rho(\rho A_\phi) - \partial_\phi A_\rho\right)\right\rangle$$

$$\langle x, y, z\rangle = \langle \rho\cos\phi \,,\ \rho\sin\phi \,,\ z\rangle \quad \ldots \ (i)$$

C. $\vec{\nabla}$ in spherical coordinates $\{r, \theta, \phi\}$

$$\langle x, y, z\rangle = \langle r\sin\theta\cos\phi \,,\ r\sin\theta\sin\phi \,,\ r\cos\theta\rangle \ \ldots \ (ii)$$

Task: Show $|\langle x, y, z\rangle|^2_{r=1} = 1$ in spherical coordinates?

$$r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta =$$

$$r^2\left(\sin^2\theta\left(\cos^2\phi + \sin^2\phi\right) + \cos^2\theta\right) =$$

$$r^2\left(\sin^2\theta + \cos^2\theta\right) = r^2\Big|_{r=1} = \underline{1}$$

$$\vec{\nabla} f = \left\langle \partial_r f \,,\ \frac{1}{r}\partial_\theta f \,,\ \frac{1}{r\sin\theta}\partial_\phi f \right\rangle$$

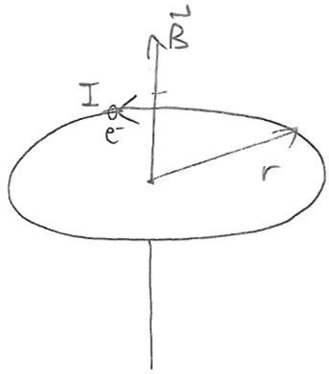
$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\partial_r(r^2 A_r) + \frac{1}{r\sin\theta}\partial_\theta(A_\theta\sin\theta)$$

$$+ \frac{1}{r\sin\theta}\partial_\phi A_\phi$$

$$\vec{\nabla} \times \vec{A} = \left\langle \begin{array}{c} \frac{1}{r \sin\theta} \left(\partial_\theta (A_\phi \sin\theta) - \partial_\phi A_\theta \right) \\ \frac{1}{r} \left(\frac{1}{\sin\theta} \partial_\phi A_r - \partial_r (r A_\phi) \right) \\ \frac{1}{r} \left(\partial_r (r A_\theta) - \partial_\theta A_r \right) \end{array} \right\rangle$$

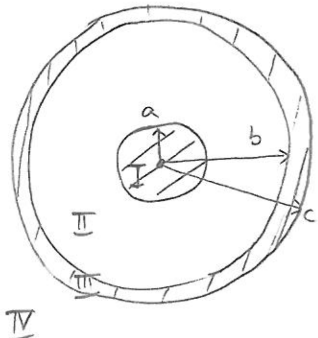
you can always use (i) & (ii) to replace $\langle A_r, A_\theta, A_\phi \rangle$, $\langle A_\rho, A_\phi, A_z \rangle$ with $\langle A_x, A_y, A_z \rangle$ & back and forth.

1.



- Calc I ?
- $\vec{B} = \dfrac{\mu_0 I}{2r}$

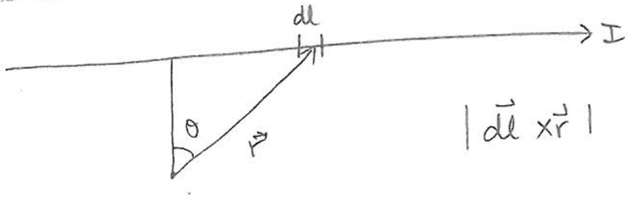
2.



@ a: Use Ampere Law: $\oint_r \vec{B}\cdot d\vec{l} = \mu_0 \int_c \vec{J}\cdot d\vec{s}$

find $\vec{B}$ in I, II, III, IV regions $= \mu_0 \int_r \left(\frac{I}{\pi a^2}\right)\cdot d\vec{s}$

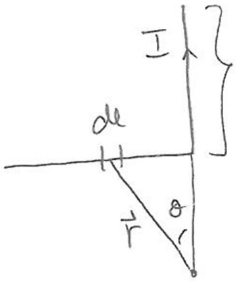
3.



$|d\vec{l} \times \vec{r}| = r\,dl\cos\theta$

a. Insert into Biot-Savart Law & integrate $\theta \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

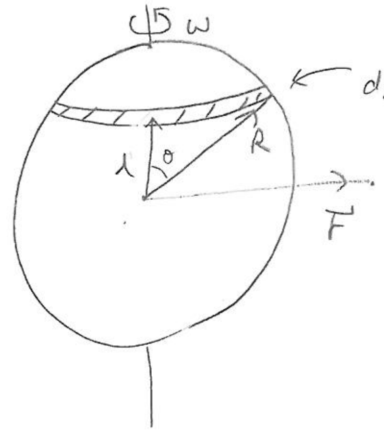
c.



This part has no contribution to $\vec{B}$!

here integration $\theta \to \left(-\frac{\pi}{2}, 0\right)$

4



$dA = dl \cdot \pi r^2$; $r^2 = R^2 - l^2$

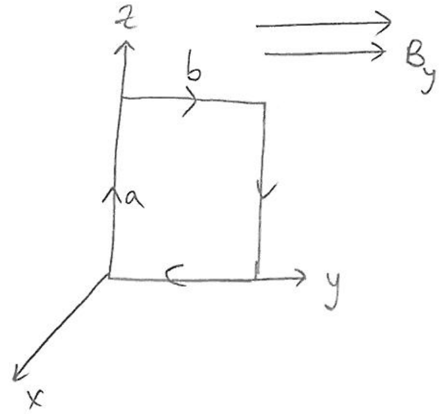
$l \to \{R, -R\}$

- Calc I, J using ω & after Integrating $\left[\frac{dA \cdot \sigma}{dt}\right]_{l \to \{R, -R\}}$

- Then use $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{j(\vec{r}')}{|\vec{r} - \vec{r}'|} \, d^3 r'$

- Then use $\vec{B} = \vec{\nabla} \times \vec{A}$

5. - Calc $\vec{F} = I(\vec{l} \times \vec{B})$ for each of 4 parts of □.



- $\vec{\mu} = I\vec{A}$
- use $\vec{\tau} = \vec{\mu} \times \vec{B}$

  $U = \vec{\mu} \cdot \vec{B}$

make sure to follow Ampere's right hand rule.