Week 7

microteaching : SR & Kinematics in particle physics

1.* SR - Transforms & notation.

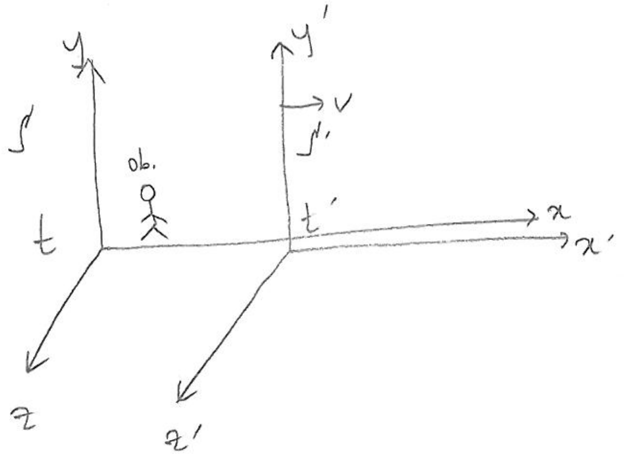
2.* Natural units

HW 7 ! — 3. ~~*#4~~ #1, #2, #3 : Kinematics equations

— 4. *#5 : Fixed Target vs. colliders

1. A. SR - Transforms of coordinates

4 Vectors : $\begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$



SR	Galile
In S : coordinates : $\langle ct, x, y, z \rangle$	$\langle ct, X, Y, z \rangle$
in S' : $t' = \gamma \left(t - \frac{vx}{c^2} \right)$	$t' = t$
$x' = \gamma (x - vt)$	$X' = X - vt$
$y' = y$	$y' = y$
$z' = z$	$z' = z$

SR | Galile

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} t' \\ x' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -v \end{bmatrix} \begin{bmatrix} t \\ x \end{bmatrix}$$

$\Lambda^{\mu}{}_{\nu}$ μ ← row, ν ← column

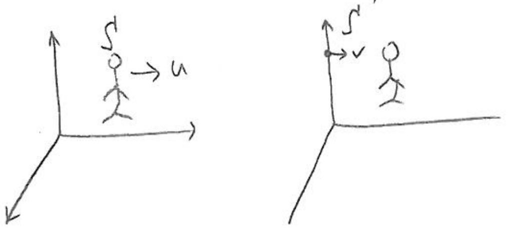
$X'^{\mu} = \Lambda^{\mu}{}_{\nu} X^{\nu}$ {Einstein notation}

$\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$; $\beta = \dfrac{v}{c}$

Task: Calc. $X'^{\mu} = {}_{v_x}\Lambda^{\mu}{}_{\nu} \; {}_{v_y}\Lambda^{\nu}{}_{\phi}$?

if $V = \langle V_x, V_y, 0\rangle$

B. Adding Velocities



SR

To S': $u' = \dfrac{u-v}{1-\frac{uv}{c^2}}$

Galile

$u' = u - v$

Task: 1. does $u' > c$ ever? ; does $u' > c$ ever

2. calc $u'(c)$? for 2 photons $\xrightarrow[c]{}$ $\xleftarrow[c]{}$? $\boxed{u' = \dfrac{c+c}{1+1} = c}$

for all moving bodies of ob. in stationary frame

I: Length contraction

$l' = \dfrac{l}{\gamma}$

II: Time dialation

$t' = \gamma t$

Mass: $m' = m\gamma$

2. Natural Units:

- So far you used SI system: $\{s, kg, m, \dots\}$
- what if I told you a unit system could be developed by setting $\underline{\hbar = c = k = 1}$?!?

- write everything in terms of Energy units ($GeV = e \times 10^9$ J)

$$g = 5.62 \times 10^{23} \text{ GeV}$$
$$K = 8.62 \times 10^{-14} \text{ GeV}$$
$$cm^{-1} = 1.98 \times 10^{-14} \text{ GeV}$$
$$s^{-1} = 6.58 \times 10^{19} \text{ GeV}$$
$$m_p = 1 \text{ GeV}$$

Task: $1 m = ?$
$1 h = ?$

$$E = mc^2 = m \text{ !!!}$$

HW 7

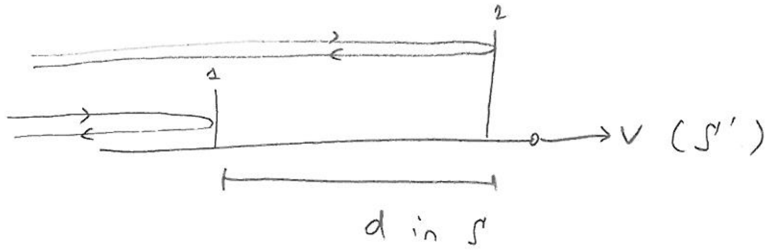
1. b. Use coordinate Transforms.

a. length contraction: $l' = \frac{l}{\gamma} = \frac{20\text{cm}}{2} \sim 10\text{cm}$

$$\gamma = \frac{1}{\sqrt{1-(\frac{.87c}{c})^2}} = 2 = \frac{1}{.5}$$

c. use adding velocities.

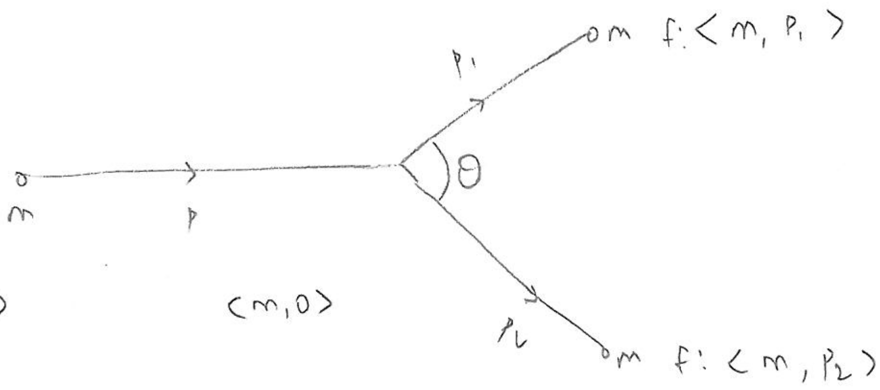
2.



$$\Delta t = \frac{2d_{eff}}{v_{eff}} = \frac{2d/\gamma}{c-v} = \frac{2d}{\gamma(c-v)}$$

3. Think about it, google if need be.

4.



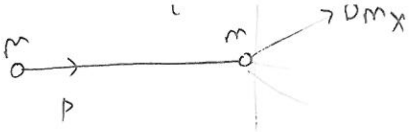
i: $\langle E_0, p\rangle$ $\langle m, 0\rangle$

Only use: 1. $p = p_1 + p_2$

$$p^2 = p_1^2 + p_2^2 + 2p_1p_2\cos\theta$$

2. $E^2 = m^2 + p^2$

5. c.



$\boxed{P \cdot P = m^2}$

i	f
$P_1 = \langle E_p, p \rangle$	$P_X = \langle E_X, P_X \rangle$
$P_2 = \langle m, 0 \rangle$	

$$P_X = P_1 + P_2 \Rightarrow P_X^2 = P_1^2 + P_2^2 + P_1 \cdot P_2 + P_2 \cdot P_1$$

$$P_X^2 = m^2 + m^2 + E_p m + m E_p = 2m^2 + 2mE_p$$

$$m_X^2 = 2m^2 + 2mE_p = 2m^2 + 2m^2\gamma$$

$$E_p = \gamma m$$

$$m_X = m\sqrt{2(1+\gamma)}$$

d.

i	f
$P_1 = \langle E_p, p \rangle$	$P_X = \langle E_X, P_X \rangle$
$P_2 = \langle E_p, -p \rangle$	

$$P_X^2 = (P_1 + P_2)^2 = (2E_p)^2 = 4E_p^2$$

$$m_X^2 = 4E_p^2 \Rightarrow m_X = 2E_p = 2\gamma m > m\sqrt{2(1+\gamma)}$$