microteaching: Kirchoff's Rules & Capacitors

1. Dimensional Analysis

Every physical quantity has units.

basic units :- time : s | Charge : c
length : m | Temp : k
mass : kg

You can get formulas upto dimensionless constants just by using units.

a. momentum depends on mass & velocity. derive relation?

$[P] = kg\,m s^{-1}$; $= (kg) \cdot (m s^{-1}) = mv$

b. Task : Energy depends on mass & velocity, derive relation, $[E] = kg\, m^2 s^{-2}$; $E = mv^2$;

really $T_{KE} = (\frac{1}{2})\, \underbrace{mv^2}$ ← has dimentions.

↑ dimensionless

A. Units multiply & devide like numbers :

find units of $F = \underbrace{k}_{k?} \cdot \frac{q \cdot Q}{r^2}$ $[k]$?

$kg\, m s^{-2} = [k] \cdot c^2 m^{-2}$

$[k] = kg\, m^3 s^{-2} c^{-2}$

B. You can only add terms with same units.

Task: ☆ Is this correct? (Look out in HW)

$$F = \frac{mv}{T} + \frac{kq}{r^2} \ ?$$

$$kg\,m\,s^{-2} = kg\,m\,s^{-2} + kg\,m\,s^{-2}\,\underbrace{C^{-1}}$$

C. Inside special functions of sin, exp, cosh, ...
Should be dimentionless terms.

$$\langle E \rangle = \frac{\partial \ln Z}{\partial\left(\frac{1}{k_B T}\right)}, \quad Z = e^{-\underbrace{\frac{E}{k_B T}}}$$

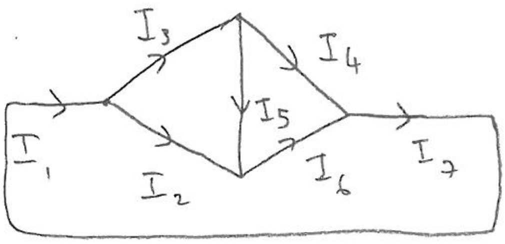
see this is dimensionless

---

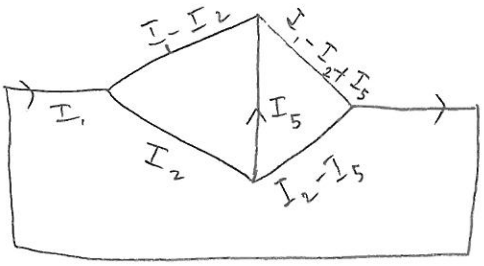
2. Kirchoff's Law

A Add currents @ junctions

Task:



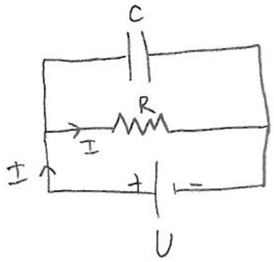
$I_1 = I_7$
$I_1 = I_3 + I_2$
$I_6 = I_5 + I_2$
$I_4 = I_3 - I_5$
$I_4 + I_6 = I_1 = I_7$



$$I_1 - \cancel{I_2} + \cancel{I_5} + \cancel{I_2} - \cancel{I_5}$$

B. Potential drops add & potential drops in loops $= 0$

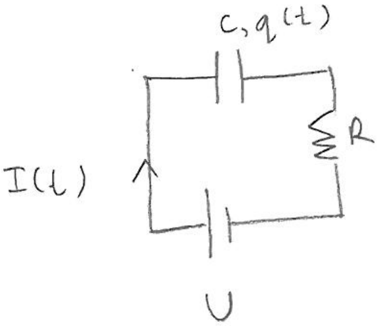
Task: <u>my</u>



$U_C = -U$

$IR = U$

3. Charging of capacitors:

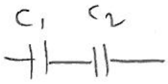
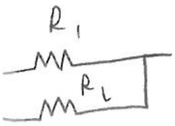
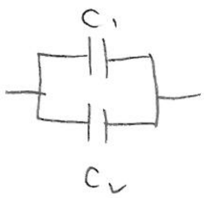


$$I(t) = \frac{dq}{dt} \; ; \quad U = IR + \frac{q}{C} = \quad U = \dot{q} R + \frac{q}{C}$$

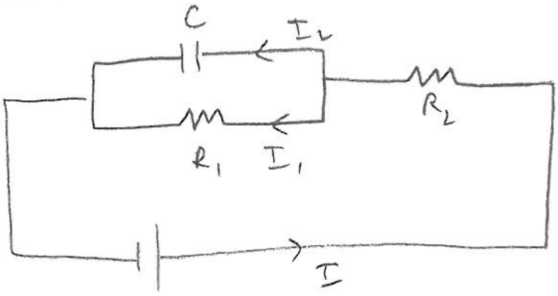
$$\dot{q} + \frac{q}{CR} = \frac{U}{R} \Rightarrow \quad q(t) = CU\left[1 - e^{-\frac{t}{RC}}\right]$$

$$I(t) = \frac{U}{R} e^{-\frac{t}{RC}}$$

4. Adding Resistors | Adding capacitors

 R_1 R_2	$R = R_1 + R_2$	 C_1 C_2	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$	Series
 R_1 R_2	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$	 C_1 C_2	$C = C_1 + C_2$	Parallel

2b.



$$U = (I_1 + I_2)R_2 + I_1 R_1 + \frac{q(t)}{C}$$

$$I_1 R_1 = I_2 \frac{q}{C} \Rightarrow I_1 = I_2 \frac{q}{CR_1}$$

$$U = I_2 \left(\frac{q}{CR_1} + R_2 \right) + I_2 \frac{q}{CR_1} + \frac{q}{CR_1}$$

$$I_2 = \frac{dq}{dt} = \dot{q} \Rightarrow \text{solve DE}$$

3. ☆ $\vec{J} = \sigma \vec{E}$ (Ohm's Law)

☆ $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$ (Conservation of charge)

☆ $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ (Gauss's Law)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\sigma \vec{E}) = -\sigma \vec{\nabla} \cdot \vec{E} = -\frac{\sigma \rho}{\varepsilon_0} \Rightarrow$$

$$\frac{\partial \rho}{\partial t} + \frac{\sigma \rho}{\varepsilon_0} = 0$$

4. Use dimentional analysis to derive relation between

J , n , q , v

$Cs^{-1}m^{-2}$, m^{-3} , C , ms^{-1} $\Rightarrow$ $J = nqv$

remember to add j^+ & j^- to calculate total 'I'.

5. $$E_c = \underbrace{\frac{1}{2} CU^2}_{} = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} QU$$

I like this

6. $$RI = \frac{Q_1}{C_1} - \frac{Q_2}{C_2}$$

Remember we used $\dot{Q} = I$, lets use

instead lets differental above equation

$$R\dot{I} = \frac{\dot{Q}_1}{C_1} - \frac{\dot{Q}_2}{C_2} = \frac{I}{C_1} - \frac{I}{C_2}$$

This is second way to setup DE in Kirchoff's Laws.

c. ☆ Energy dissipated through 'R'

$\Downarrow$

$$E_r = IR^2 = \frac{V^2}{R} = VI.$$