Mar 16, 2016

micro-teaching : Scalars & Vector fields

Scalar & Vector fields

+ operations on scalar & vector fields.

1. scalar fields
2. Vector fields

— — — Operations ↓

3. Divergence
4. Gradient
5. Curl

All fields : Some function : $f(x)$

f : scalar / vector

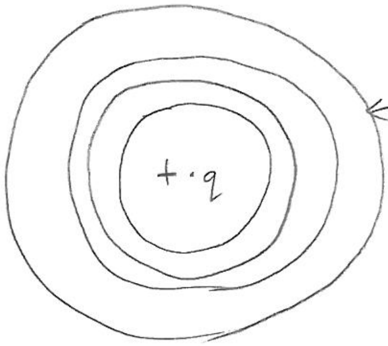
x : scalar / vector (space-coordinate)

1. Scalar fields.

Potential : $V(\vec{x})$

V : scalar

$\vec{x}$: Vector.



Equipotential surface where $V_i(\vec{x}_i) = V_j(\vec{x}_j)$

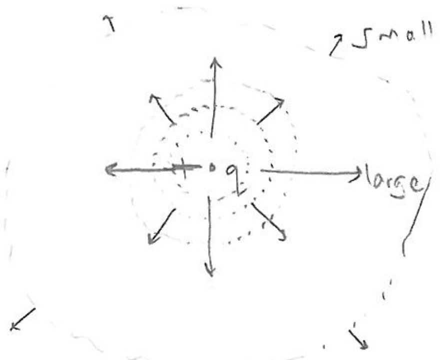
2. Vector fields:

eg. field (Electric) :

$\vec{E}(\vec{X})$

$\vec{E}$: Vector

$\vec{X}$: Vector



Operations :

$$\underbrace{\vec{\nabla}\ ,\ \vec{\nabla}\cdot\ ,\ \vec{\nabla}\times}\ ,\ \nabla^2\ ,\ \Box^2\ \ldots$$

3 basic ones , $\vec{\nabla} = [\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}]$

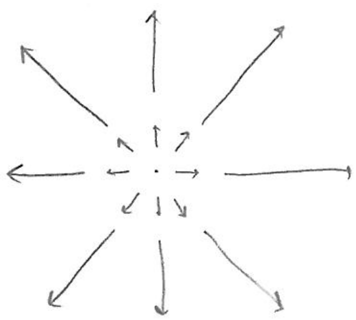
$\underbrace{\text{gradient , divergence , Curl}}$, Laplacian , d'Alembertian

3. Divergence : $\underbrace{\vec{\nabla}\cdot\vec{E}}_{\text{scalar}} = \frac{\rho}{\varepsilon_0}$ (Gauss' Law)

vectors

Divergence gives you a sense of how much of vector field's ($\vec{E}$) magnitude is being created or destroyed @ each point in space (i.e. sink & sources)

eg. : Calc divergence of this field



$\vec{E}_1 = \langle 2x, 2y, 0 \rangle$

$\vec{\nabla}\cdot\vec{E}_1 = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \cdot \langle 2x, 2y, 0 \rangle$

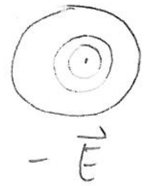
$= 4$

4. Gradient : $\underbrace{\vec{\nabla}V}_{\text{vector}} = -\vec{E}$

scalar

Just 3D vector version of derivative.

eg: Calc gradient of: $V(\vec{x}) = (x^2+y^2+z^2)^{-\frac{1}{2}}$



$$\vec{\nabla} V = -\frac{1}{2V^{\frac{3}{2}}}\langle 2x, 2y, 2z\rangle = -\langle x, y, z\rangle \frac{1}{\sqrt{(x^2+y^2+z^2)^3}} = -\vec{E}$$

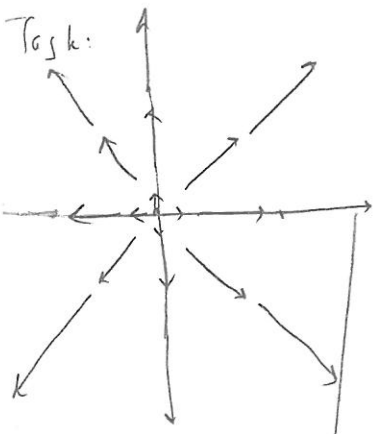
5. Curl: $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$

vectors

Gives a sense of how much twist exists in a vector field. (neglects change in magnitude like that picked out by divergence)

eg. Calc. $\nabla\times\langle -y, x, 0\rangle = \langle 0, 0, 2\rangle$

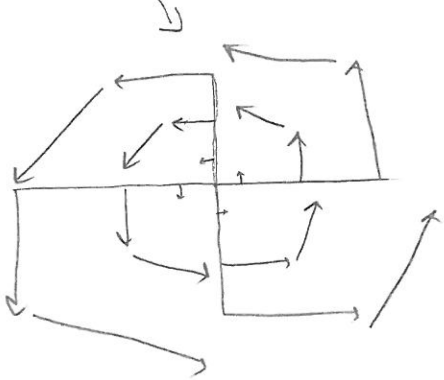
Task:



$E_1 = \langle 2x, 2y, 0\rangle$

$\vec{\nabla}\cdot\vec{E}_1 = ? = 4$

$\vec{\nabla}\times\vec{E}_1 = ? = 0$



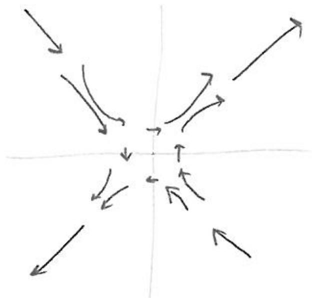
$E_2 = \langle -y, x, 0\rangle$

$\vec{\nabla}\cdot\vec{E}_2 = 0$

$\vec{\nabla}\times\vec{E}_2 = \langle 0, 0, 2\rangle$

$|\nabla\times\vec{E}_2| = 2 \neq 0$

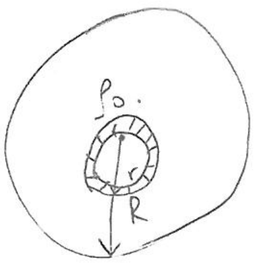
Not Req.



$E_3 = \langle y, x, 0\rangle$

HW4: Electrostatics. Prajwol : 4

1.



$\rho(\vec{r}) = \rho_0 \quad \{0 < |\vec{r}| < R\}$

a. $q = \int \rho_0 \, dV$; $dV = r^2 \sin^2\theta \, dr \, d\phi \, d\theta$, $\theta \to (0, \pi)$

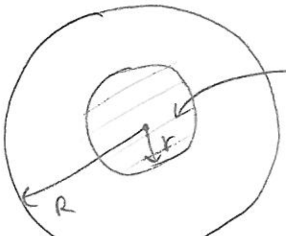
$\sim 4 r^2 \pi \, dr$ $\quad \phi \to (0, 2\pi)$

$r \to (0, R)$

b. for $0 < r < R$ $\ell \to$? make sure to use only q enclosed in 'r'

$R < r < \infty \longrightarrow k_0 \cdot \frac{q}{r^2}$

$0 < r < R$:



$q_r = \int_0^r \rho_0 \, 4 r^2 \pi \, dr = \oint \vec{E} \cdot d\vec{a}$

write $E_1 (0 < r < R)$, $E_2 (R < r < \infty)$ } find value of $V = -\int E \cdot dl$

c. . . .

$|\vec{E}| = |E_r \cdot \hat{r}| = E_r$

d. $dU = V \cdot dq \leftarrow dq = \rho_0 \cdot 4\pi r^2 dr$, $\int_0^R dU$?

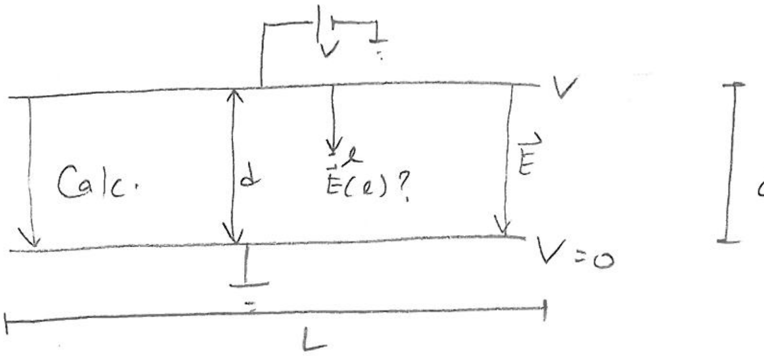
from b.

e. Calc. $\int_0^R dr \cdot \frac{\varepsilon_0}{2} E^2(r) \cdot 4\pi r^2 = ?$

integrals are not simple but do them...

2. Prajwal:5

a. $\xrightarrow[v]{m,q}$



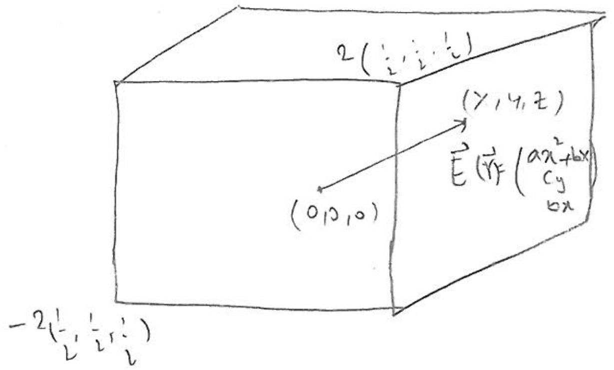
i. Calc. acceleration $a = \frac{E}{m} = \frac{Eq}{m}$

ii. Calc displacement, x : $x = \frac{1}{2}at^2$; $t = \frac{v}{L}$

b. find max v. by setting $x \to d$

All relation in above steps

3.



a. $\vec{\nabla} V = -\vec{E} \Rightarrow \frac{\partial V}{\partial x} = -ax^2 - bx$; $\frac{\partial V}{\partial y} = -cy$; $\frac{\partial V}{\partial z} = -bx$

b. i. Calc $F = Eq$

ii. $W = \int_{\langle -1,-1,-1 \rangle}^{\langle 1,1,1 \rangle} F \cdot dr$

c. $\vec{F} = \vec{\nabla}(\vec{p} \cdot \vec{E})$

$\vec{\tau} = \vec{p} \times \vec{E}$

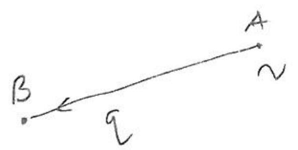
Prajwal : 6

d. $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow$

i. $\rho = (\vec{\nabla} \cdot \vec{E})\, \varepsilon_0$?

ii. $\int d\rho$?

Extra 1: Work Energy Theorem.



final, initial

$$W_{AB} = V_B - V_A = \int_{r_A}^{r_B} F\,dr = \int_{r_A}^{r_B} E(r)\,dr$$

Extra 2: Conservation of Energy :

$$KE_i + PE_i = KE_f + PE_f$$

PE :
- Potential from pot. field : $PE = Vq$
- from $\vec{E}$ field : $\int \frac{\varepsilon_0 E^2}{2}\, dV$
- from $\vec{B}$ field : $\int \frac{B^2}{\mu_0 2}\, dV$
- from grav potential : mgh (@ height h)

KE : $\frac{1}{2} m v^2$