Week 3: Partial Differential Eq.

Thursday, March 10, 2016 9:27

Wave Equation:

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial \psi}{\partial x^2}$$

Let $\psi = X(x)\,T(t)$

Seperation of Variables:

$$X(x)\,\ddot{T}(t) = v^2\,\ddot{X}(x)\,T(t)$$

$\div\ X(x)\,T(t)$

$$\frac{\ddot{T}(t)}{T(t)} = -\lambda = \frac{v^2\,\ddot{X}(x)}{X(x)}$$

1. $\ddot{T}(t) + \lambda T(t) = 0$

$$\Gamma(t) \sim e^{mt}$$

$$m^2 - \lambda = 0 \Rightarrow \Gamma(t) = c_1 e^{i\sqrt{\lambda}t} + c_2 e^{i\sqrt{\lambda}t}$$

2 $$v^2 \ddot{X}(x) - \lambda X(x) = 0$$

$$X(x) = e^{nx}$$

$$v^2 n^2 - \lambda = 0 \Rightarrow X(x) = b_1 e^{i\frac{x}{v}\sqrt{\lambda}} + b_2 e^{-i\frac{x}{v}\sqrt{\lambda}}$$

$$\text{let } \sqrt{\lambda} = \omega \Rightarrow \frac{\sqrt{\lambda}}{v} = \frac{\omega}{v} = k$$