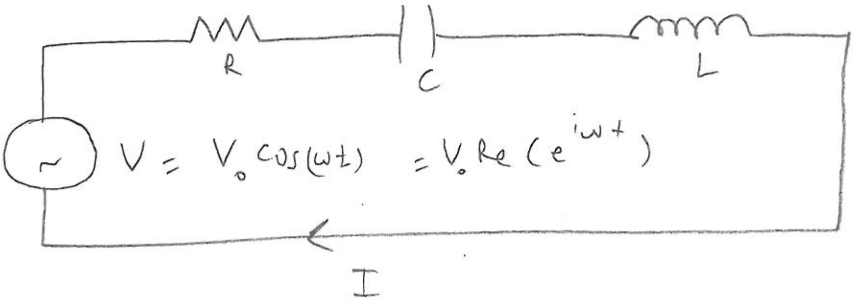
Week 11

micro teaching : AC circuits



individual impedence :

Re	Re+Im
R	R
$1/\omega C$	$(-i/\omega C)\, e^{-\frac{i\pi}{2}}$
ωL	$(i\omega L)\, e^{\frac{i\pi}{2}}$

Total impedence : $R + i\omega L - \frac{i}{\omega C} = Z$

$$: \sqrt{R^2 + \omega^2 L^2 + \frac{1}{\omega^2 c^2}} = \mathrm{Re}(Z)$$

1. Let $(L=0)$

$$V_R = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}} V_0 \cos(\omega t)$$

$$V_C = \frac{1/\omega c}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}} \cdot V_0 \cos(\omega t - \tfrac{\pi}{2})$$

Lags by $\pi/2$

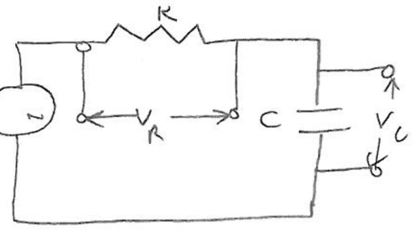
2. Let $(C=0)$ Task

$$V_R = ? = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t)$$

$$V_C = ? = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \tfrac{\pi}{2})$$

In an RC circuit:

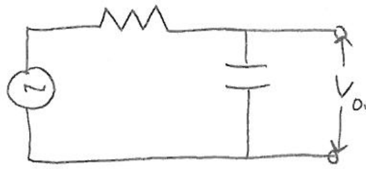
HP / LP filter



Task

1. Notice constant $I(\omega t)$ doesn't pass through C

In.

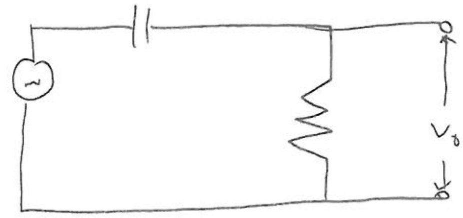


$$V_{out} \propto \frac{1/\omega C}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = A = \frac{1}{\sqrt{R^2 \omega^2 C^2 + 1}}$$

$\Rightarrow |V_{out}| = V_0 A \Rightarrow$ Low pass $(\omega \to 0) \Rightarrow (A = 1)$

2. Notice all $I(\omega t)$ passes through R

in.



$$V_{out} \propto \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{R\omega C}{\sqrt{R^2 \omega^2 C^2 + 1}}$$

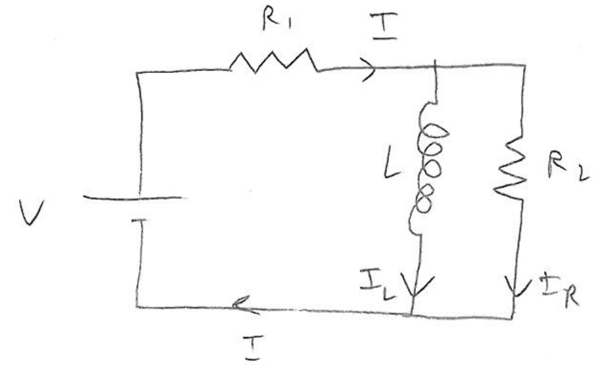
$\Rightarrow |V_{out}| = V_0 A \Rightarrow$ High pass

$(\omega \to \infty) \Rightarrow A = 1$

$\Rightarrow V_C \Rightarrow$ LP | $V_R \Rightarrow$ HP

HW 11

1.



a. $* \; I = I_L + I_R \; ; \quad V - IR_1 - L\dot{I}_L = 0$

$$\dot{I}_L L - I_R R_2 = 0$$

$$V - IR_1 - I_R R_2 = 0$$

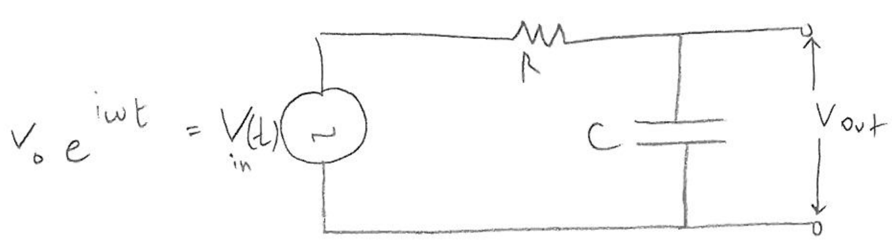
$*' \Rightarrow$ Calc. $I_L(t)$ & $I_R(t)$ using $I_L(0) = 0$

$$I_L(\infty) = \frac{V}{R_1}$$

$*$ Calc. $\int_0^\infty I_R^2(t) R_2 \, dt = ?$

b. hint : what energy is stored in 'L' $= \frac{1}{2} L I_L^2(\infty)$

2



$$Z = R - \frac{i}{\omega C}$$

a. $I(t) = \frac{V_{in}}{Z}$

b. $V_{out} = \frac{Z_c}{Z} V_{in}(t)$

c. Easy way: $I = Z_c \frac{V_{in}(t)}{Z_r} = \frac{\frac{1}{\omega c} V_{in}}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}} = \frac{V_{in}}{\sqrt{1 + R^2 \omega^2 c^2}}$

$\tan \delta = \frac{Im(Z_c/Z)}{Re(Z_c/Z)}$; complex part only usefl for phase info

d, e. Not a big change.

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3. a. $\vec{B} = ?$ Obtain $\vec{E}$ from: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Should have same phase info.

b. $T \sim \underbrace{[0, 2\pi]}_{(kx - \omega t)}$ → do not complicate things

c. $P = \frac{2}{c} |\langle S \rangle|$ assuming photons bounce off elastically.

$\vec{F} = \vec{A} \cdot P$

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4. a. $\vec{E} = -\frac{V}{d}(1 - e^{-\frac{t}{RC}}) \hat{z}$

according to Ampere's Law: $\vec{\nabla} \times \vec{B} = \mu_0 J = 0$?

Maxwell correction: $\vec{\nabla} \times \vec{B} = \mu_0 (J + \varepsilon_0 \frac{\partial E}{\partial t})$

$$I_{dc} = \int J_{disp.\ current} \cdot ds = \int \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \cdot ds = \int \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} \cdot ds = \frac{1}{\mu_0} \int \vec{B} \cdot d\vec{l} = \frac{2\pi r B}{\mu_0}$$

because of × product : $\vec{B}$ cannot have direction of $\hat{z}$ $\Rightarrow$ $\hat{\phi}$

b. ... $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

c. express $\vec{j}$ in terms of $V_c = V_0 (1 - e^{-\frac{t}{RC}})$ f

then $\quad W = \int dt \left(\int S(t) \Big|_{r = R_0} ds \right)$

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5. $I(-\frac{d}{2}, t) = I(\frac{d}{2}, t) = 0 \Rightarrow I$? linear in z

time part $\sim \cos(\omega t)$

eq. of continuity : $\frac{d\lambda}{dt} + \frac{dI}{dz} = 0 \Rightarrow$ obtain $\lambda(z,t)$

$$p(t) = \int_{-d/2}^{d/2} z\, \lambda(t)\, dz = ?$$

then calc. $\quad \langle P \rangle = \frac{\omega^2 p_0^2}{12\pi \varepsilon_0 c^3}$