

Week 5: Electrostatics Primer.

{ Electro magnetism Primer }
week 10

1. \vec{E} : Gauss's Law

2. V

3. C

1. \vec{E} , Gauss's Law

Electric field: $\vec{E} \equiv$ force per unit charge (+ve)

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \hat{r}$$



$$\vec{F} = \vec{E} q_2$$

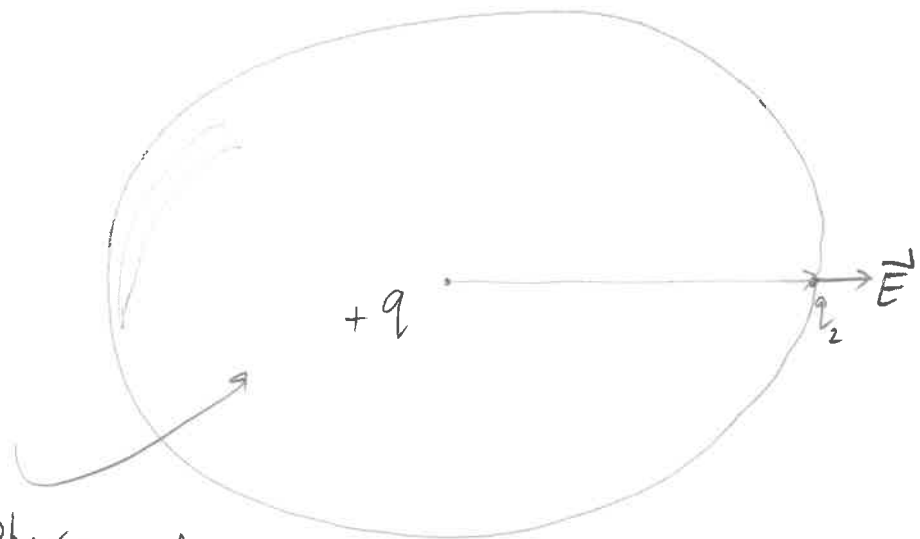
Gauss's Law: how do we get \vec{E} from 'q'

A. $\int \vec{E} \cdot d\vec{a} = \frac{q}{\epsilon_0}$

B. $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Same.

charge density



Sphere of radius - r

hard to see differential form work here
 use integral form of Gauss's Law

@ r_2 :
$$\int \vec{E} \cdot d\vec{a} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

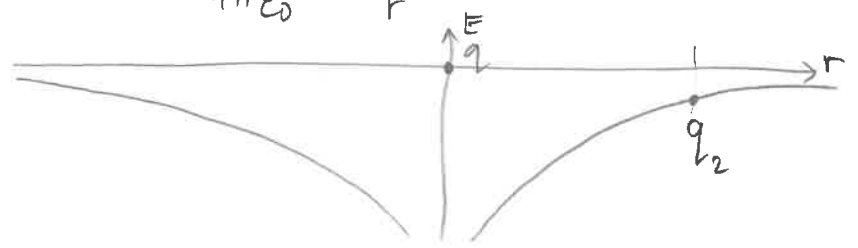
$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{\epsilon_0} \frac{q}{4\pi r^2} ; \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \hat{r}$$

2. V : Electric scalar potential.

Electric potential - $V \equiv$ Energy per unit charge (J/C)

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} ; \quad E = V_{,2}$$



It takes Energy to put 2 +ve charges together. $\equiv E = V \cdot q$

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad ; \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

A. $\int_{\infty}^r \vec{E} \cdot d\vec{r} = V_r$

B. $-\vec{\nabla}V = \vec{E}$

Same

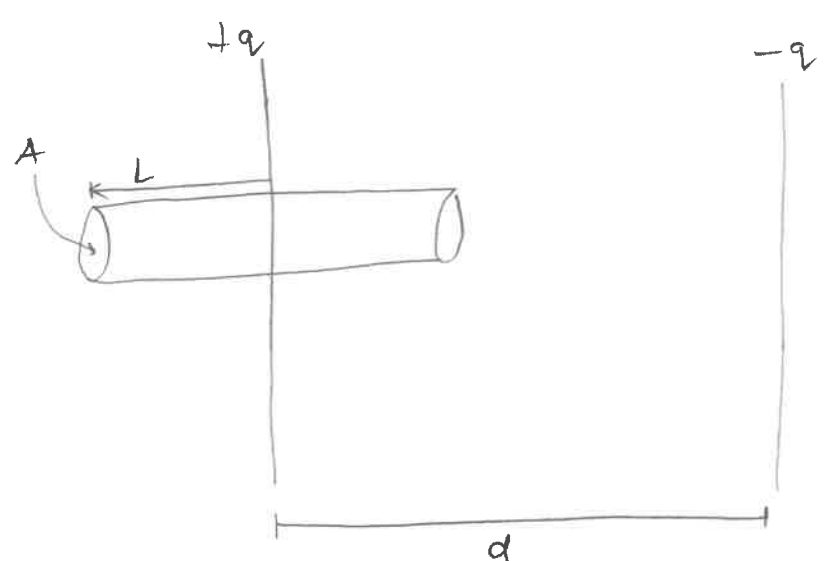
3. Capacitance: C

\equiv proportionality between charge stored for applied potential

$$q = C \cdot V$$

$$C = \frac{q}{V}$$

So what is capacitance between 2 plates?



$$\vec{E} \rightarrow V \rightarrow C$$

$$a. \vec{E} \cdot 2A = \frac{\rho \cdot A}{\epsilon_0} \rightarrow E = \frac{\rho}{2\epsilon_0}; \quad E = \frac{-\rho}{2\epsilon_0}; \quad E = \frac{\rho}{\epsilon_0}$$

$$b. V = \int E \cdot dr = \int_0^d \frac{\rho}{\epsilon_0} \cdot dr = \frac{\rho d}{\epsilon_0}$$

c. C: we need to know charge $\equiv q = \rho A$

$$C = \frac{q}{V} = \frac{\rho A}{\left(\frac{\rho d}{\epsilon_0}\right)} = \epsilon_0 \frac{A}{d}$$

1.



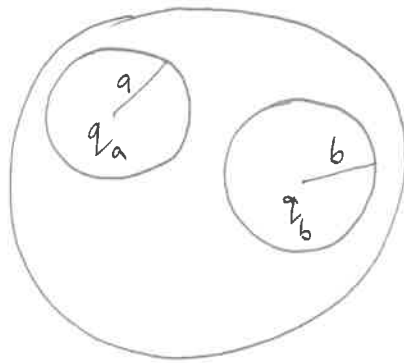
Notice : $Q_1 + Q_2 = Q_1' + Q_2'$

$V_1' = V_2'$

2 Variables: Q_1', Q_2'

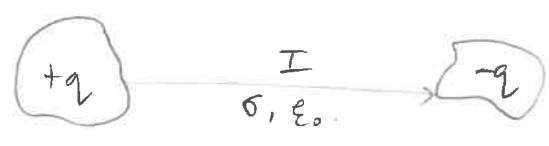
2 Eq : \rightarrow solve for $Q_1' (Q_1, Q_2)$
 $Q_2' (Q_1, Q_2)$

2.



a. $\int_{a,b} \rightarrow q_a, q_b, a, b$: use Gauss's Law

c. $\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \hat{r}$

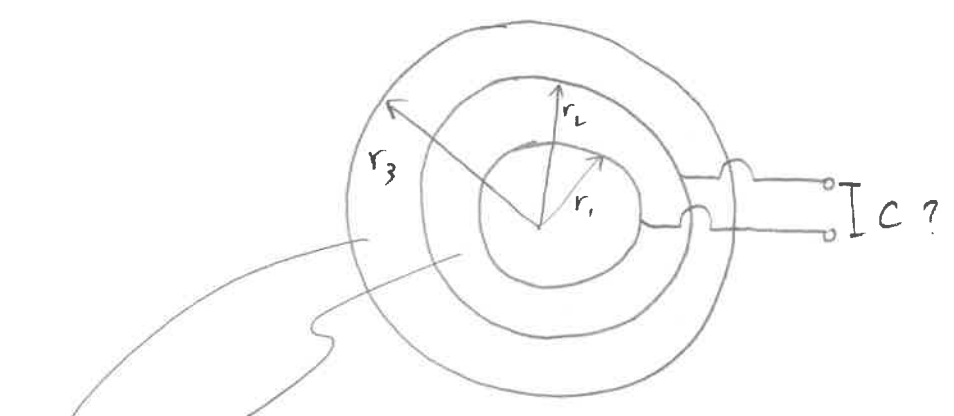


a. Use $\vec{J} = \sigma \vec{E} = \frac{1}{R} \cdot \vec{E}$

$q = CV$ { + you always have Gauss' }
 $\int E \cdot da = \frac{q}{\epsilon_0}$

c. $I = \frac{\sigma q}{\epsilon_0} = \frac{dq}{dt}$

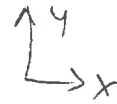
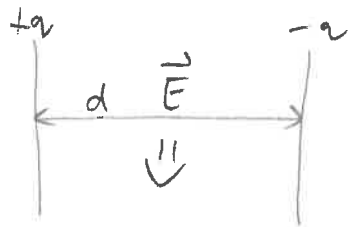
$R = \frac{\epsilon_0}{\sigma C} = \frac{V}{I} \Rightarrow I = \frac{V \sigma C}{\epsilon_0} = \frac{q \sigma}{\epsilon_0}$



$E_{1,2} = \frac{\sigma_1 r_1}{\epsilon_0 r} \rightarrow V_{1,2} = \int E \cdot dr$

$E_{2,3} = \frac{\sigma_1 r_1 + \sigma_2 r_2}{\epsilon_0 r} \rightarrow V_{2,3} = ?$

$\Delta V = ? \rightarrow C$



$$V_{\max} = E \cdot d \quad ; \quad C = \frac{\epsilon_0 A}{d}$$

you know: $q = CV$

Notice $V = E \cdot x$

$$\int U = \frac{1}{2} CV^2$$