HW-PSet: 4

Micro-teaching: Scalars & Vector fields.

Scalar & Vector fields

\+ operations on scalar & vector fields.

1. scalar fields
2. Vector fields

--- Operations ↓

3. Divergence
4. Gradient
5. Curl

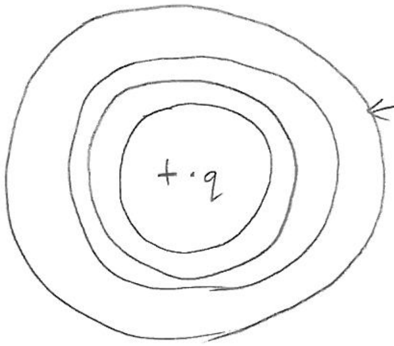
All fields : Some function : $f(x)$

$\underbrace{\text{Scalar / Vector}}$ $\leftarrow f$; $x \leftarrow$ Scalar / $\underline{\text{Vector}}$ (space-coordinate)

. Scalar fields.

Potential : $V(\vec{x})$

V: scalar ; $\vec{x}$: Vector.



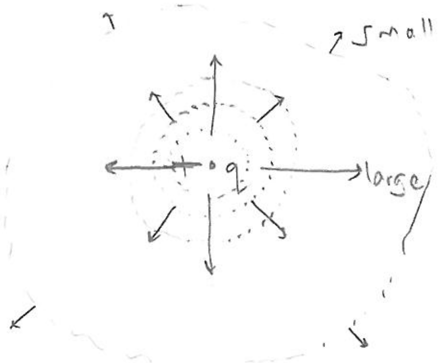
Equipotential surface where $V_i(\vec{x}_i) = V_j(\vec{x}_j)$

2. Vector fields:

eg. field (Electric) :

$\vec{E}(\vec{x})$

$\vec{E}$: Vector ; $\vec{x}$: Vector



Operations :

$$\underbrace{\vec{\nabla}\ ,\ \vec{\nabla}\cdot\ ,\ \vec{\nabla}\times}_{\text{3 basic ones}}\ ,\ \nabla^2\ ,\ \Box^2\ \ldots$$

$$\vec{\nabla} = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]$$

gradient , divergence , Curl , Laplacian , d'Alembertian

3. Divergence :

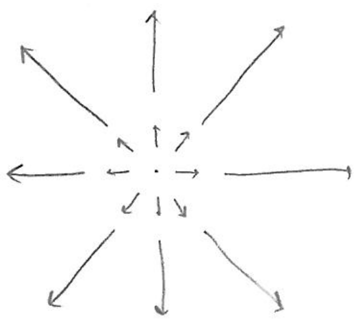
vectors

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{(Gauss' Law)}$$

scalar

Divergence gives you a sense of how much of vector field's ($\vec{E}$) magnitude is being created or destroyed @ each point in space (i.e. sink & sources)

eg. : Calc divergence of this field



$$\vec{E}_1 = \langle 2x, 2y, 0 \rangle$$

$$\vec{\nabla}\cdot\vec{E}_1 = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle 2x, 2y, 0 \rangle$$

$$= 4$$

4. Gradient :

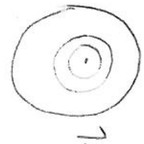
scalar

$$\vec{\nabla} V = -\vec{E}$$

vector

Just 3D vector version of derivative.

eg : Calc gradient of : $V(\vec{x}) = (x^2+y^2+z^2)^{-\frac{1}{2}}$



$$\vec{\nabla} V = -\frac{1}{2V^{\frac{3}{2}}}\langle 2x, 2y, 2z\rangle = -\langle x, y, z\rangle \frac{1}{\sqrt{(x^2+y^2+z^2)^3}} = -\vec{E}$$

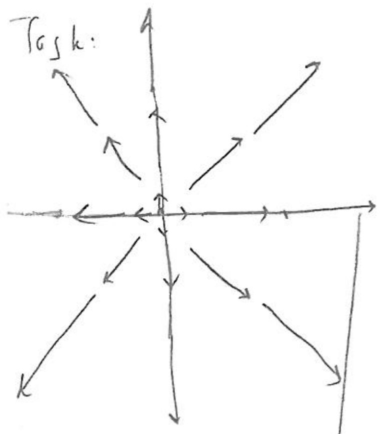
5. Curl : $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$

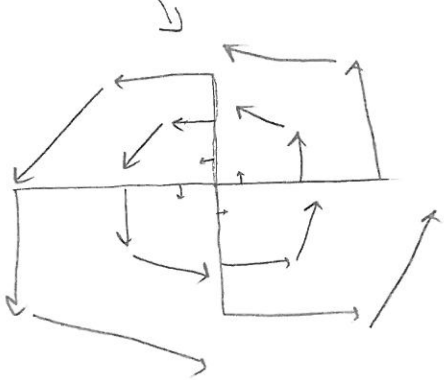
Vectors

Gives a sense of how much Twist exists in a vector field. (neglects change in magnitude like that picked out by divergence)

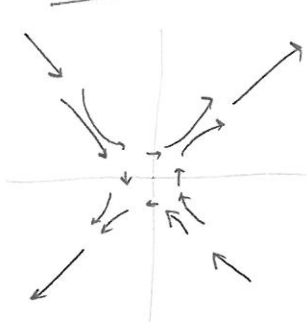
eg. Calc. $\nabla\times\langle -y, x, 0\rangle = \langle 0, 0, 2\rangle$

Task:





Not Req.



$E_1 = \langle 2x, 2y, 0\rangle$

$\vec{\nabla}\cdot\vec{E}_1 = ? = 4$

$\vec{\nabla}\times\vec{E}_1 = ? = 0$

$E_2 = \langle -y, x, 0\rangle$

$\vec{\nabla}\cdot\vec{E}_2 = 0$

$\vec{\nabla}\times\vec{E}_2 = \langle 0, 0, 2\rangle$

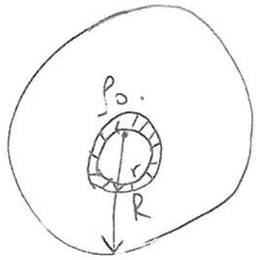
$|\nabla\times\vec{E}_2| = 2 \neq 0$

$E_3 = \langle y, x, 0\rangle$

HW4: Electrostatics

Prajwol : 4

1.



$\rho(\vec{r}) = \rho_0 \quad \{0 < |\vec{r}| < R\}$

a. $q = \int \rho_0 \, dv$; $dv = r^2 \sin^2\theta \, dr \, d\phi \, d\theta$, $\theta \to (0, \pi)$

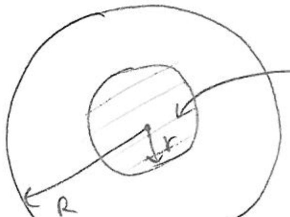
$\sim 4 r^2 \pi \, dr$ $\qquad \phi \to (0, 2\pi)$

$r \to (0, R)$

b. for $0 < r < R$? $\to$? make sure to use only q enclosed in 'r'

$R < r < \infty \longrightarrow k_0 \cdot \frac{q}{r^2}$

$0 < r < R$:



$q_r = \int_0^r \rho_0 \, 4 r^2 \pi \, dr = \oint \vec{E} \cdot d\vec{a}$

write $E_1 (0 < r < R)$
$E_2 (R < r < \infty)$ } find value of $V = -\int E \cdot dl$

c. . . .

$|\vec{E}| = |E_r \cdot \hat{r}| = E_r$

d. $dU = V \cdot dq \leftarrow dq = \rho_0 \cdot 4\pi r^2 dr$, $\int_0^R dU$?

from b.

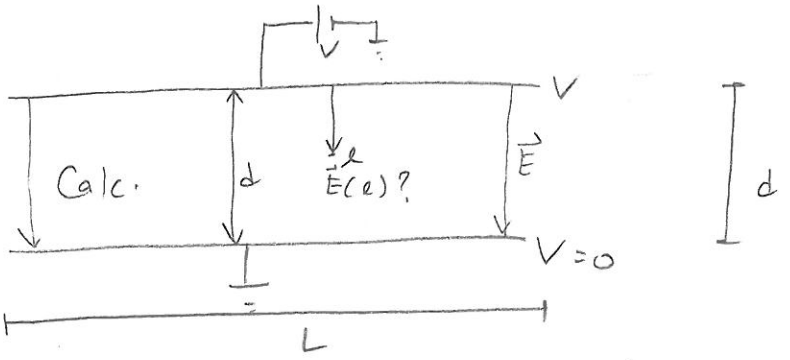
e. Calc. $\int_0^R dr \cdot \frac{\epsilon_0}{2} E^2(r) \cdot 4\pi r^2 = ?$

Integrals are not simple but do them ...

2.

Prajwal:5

a. $\xrightarrow{m,q}$ v



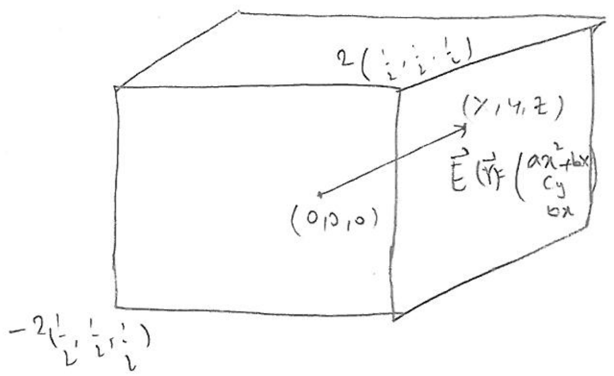
i. Calc. acceleration $a = \frac{E}{m} = \frac{Eq}{m}$

ii. Calc displacement, x : $x = \frac{1}{2}at^2$; $t = \frac{v}{L}$

b. find max v. by setting $x \to d$

All relation in above steps

3.



a. $\vec{\nabla} V = -\vec{E} \Rightarrow \frac{\partial V}{\partial x} = -ax^2 - bx$; $\frac{\partial V}{\partial y} = -cy$; $\frac{\partial V}{\partial z} = -bx$

b. i. Calc $F = Eq$

ii. $W = \int_{\langle -1,-1,-1 \rangle}^{\langle 1,1,1 \rangle} F \cdot dr$

c. $\vec{F} = \vec{\nabla}(\vec{p} \cdot \vec{E})$

$\vec{\tau} = \vec{p} \times \vec{E}$

Projwal: 6

d. $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow$

i. $\rho = (\vec{\nabla}\cdot\vec{E})\,\varepsilon_0$?

ii. $\int d\rho$?

<u>Extra1</u>: Work Energy Theorem.

final

initial

$$W_{AB} = V_B - V_A = \int_{r_A}^{r_B} F\,dr = \int_{r_A}^{r_B} E(r)\,dr$$

<u>Extra 2</u>: Conservation of Energy:

$$KE_i + PE_i = KE_f + PE_f$$

PE:
- Potential from pot. field : $PE = Vq$
- from $\vec{E}$ field : $\int \frac{\varepsilon_0 E^2}{2}\,dV$
- from $\vec{B}$ field : $\int \frac{B^2}{\mu_0 2}\,dV$
- from grav potential : mgh (@ height h)

KE: $\frac{1}{2}mv^2$