Week 3: Partial Differential Eq.

Wave Equation:

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial \psi}{\partial x^2}$$

dimensionally it should be some velocity

let $\psi = X(x)\, T(t)$

Seperation of Variables.

$$X(x)\, \ddot{T}(t) = v^2 \ddot{X}(x)\, T(t)$$

$\div \; X(x)\, T(t)$

$$\therefore \quad \frac{\ddot{T}(t)}{T(t)} = -\lambda = \frac{v^2 \ddot{X}(x)}{X(x)}$$

1. $\ddot{T}(t) + \lambda T(t) = 0$

$$T(t) \sim e^{mt}$$

$$m^2 - \lambda = 0 \Rightarrow T(t) = c_1 e^{i\sqrt{\lambda}t} + c_2 e^{i\sqrt{\lambda}t}$$

2 $$v^2 \ddot{X}(x) - \lambda X(x) = 0$$

$$X(x) = e^{nx}$$

$$v^2 n^2 - \lambda = 0 \Rightarrow X(x) = b_1 e^{i\frac{x}{v}\sqrt{\lambda}} + b_2 e^{-i\frac{x}{v}\sqrt{\lambda}}$$

$$\text{let } \sqrt{\lambda} = \omega \Rightarrow \frac{\sqrt{\lambda}}{v} = \frac{\omega}{V} = k$$

General solution to wave equation: $A\cos(kx - \omega t)$

Plug in: $$\partial_t^2 \psi = \omega^2 A\cos(kx - \omega t)$$

$$\partial_x^2 \psi = k^2 A\cos(kx - \omega t)$$

$$\frac{\partial_t^2 \psi}{\partial_x^2 \psi} = \frac{\omega^2}{k^2} = \lambda^2 f^2 = V^2$$

Electrodynamics: Maxwell's Laws for Empty space

Gauss: $\nabla \cdot E = 0$

no-name: $\nabla \cdot B = 0$

Faraday's: $\nabla \times E = -\frac{\partial B}{\partial t}$

Ampere's: $\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$

Permeability (B)

Permitivity (E)

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times (\nabla \times E) = -\nabla \times \frac{\partial B}{\partial t}$$

$$\nabla(\underbrace{\cancel{\nabla \cdot E}}_{\text{Gauss}}{}^{0}) - \nabla^2 E = -\frac{\partial}{\partial t}\underbrace{(\nabla \times B)}_{\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}}$$

Use: $[\nabla \times \nabla \times \psi = \nabla(\nabla \cdot \psi) - \nabla^2 \psi]$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

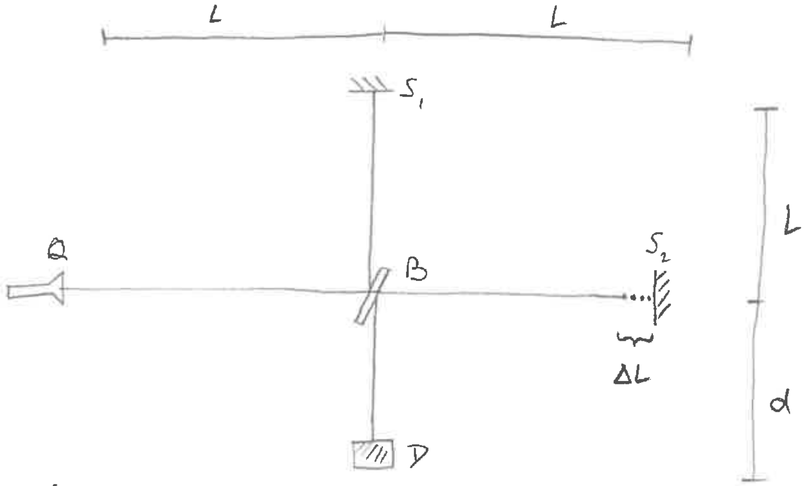
In 1D: $\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$

$$\Rightarrow c_0^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$! EM waves.

HW-3

1.



a. $Q = A\cos(kx - \omega t)$

$D = A\cos(k\Delta L)\cos(k(b+2L+d+\Delta L) - \omega t)$

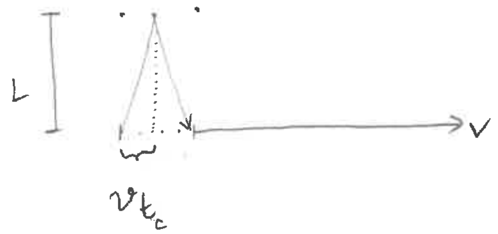
$\delta x = 2\Delta L$

b. remember $I \sim A^2$

constructive $\Delta x \sim \frac{n\lambda}{2}$,

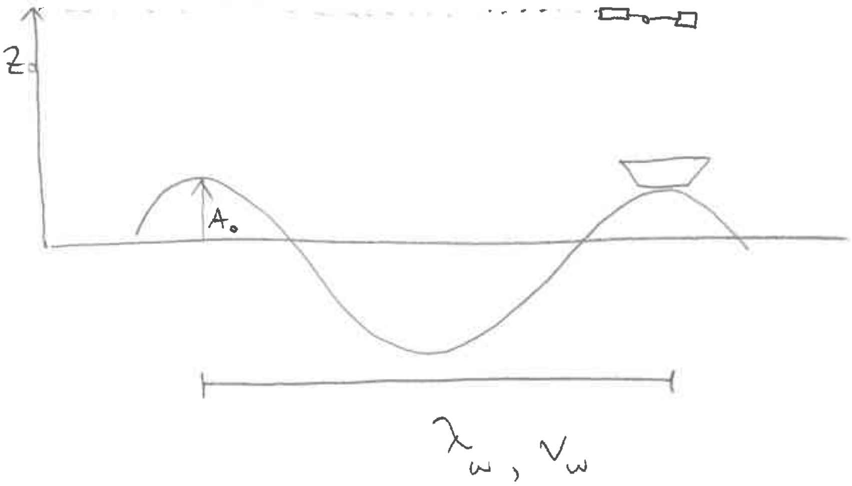
destructive : $\Delta x \sim (\frac{n}{2}+1)\lambda$

c. Use pythagorean theorem: if motion is $\perp$ to $\vec{v}$



$ct_c = \sqrt{L^2 + (vt_c)^2}$

2.



a. Consider it a standing wave

b; c; doppler effect

$$y(t) = A_o \cos(\omega t)$$ from v, λ

$$v_b = \frac{dy(t)}{dt}$$

$$f_{satellite} = \frac{f_{radio}}{1 - \frac{v_b}{c}}$$

d. Look @ HW2 - #4

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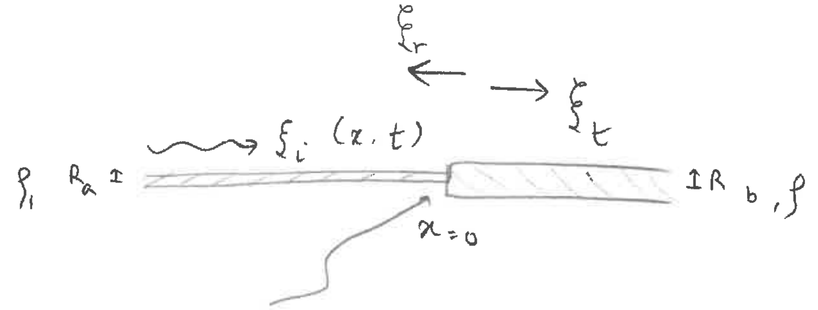
3. $v = \lambda f \Rightarrow v = \frac{\lambda}{T}$

Notice orbital velocity $= v = \sqrt{gh}$ where 'h' is height.

so $\Rightarrow \frac{\lambda}{T} = \sqrt{gh} \Rightarrow h = \frac{\lambda^2}{T^2 g}$

you are given $T \sim 25h$; & you need λ, but you are also given $L \sim 1200 km$..., so figure out what λ to use.

4.



a.

soft reflection, so no 'π' phase difference

write down ξ_i & ξ_r, ξ_t

b.

boundary conditions:

position $\qquad \xi_i(0,t) + \xi_r(0,t) = \xi_t(0,t)$

★ wave Equation: $\dfrac{\partial \xi_i(0,t)}{\partial x} + \dfrac{\partial \xi_r(0,t)}{\partial x} = \dfrac{\partial \xi_t(0,t)}{\partial x}$

c. $\quad$ Use $\quad v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{F}{\rho \pi R^2}}$ $\quad$ This constrains V_a & V_b .. $F_a = F_b$

d. $\quad I = \frac{1}{2} \rho v \omega^2 A^2$

$\omega \to 2\pi f$