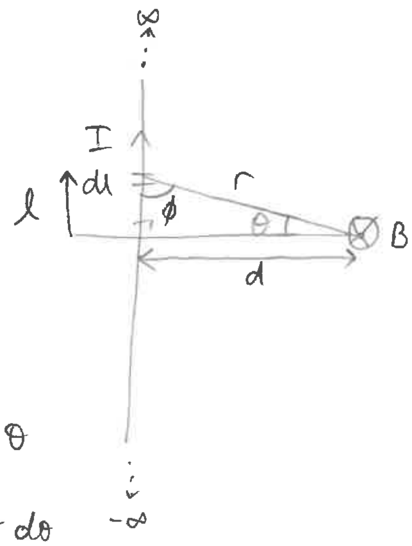
Week 10: Primer on Magnetostatics: $\vec{B}$

1. Biot-Savart's Law:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times d\vec{l}}{r^2}$$

<u>Task</u>: Infinite wire: $B(d)$?



$$B = \frac{\mu_0}{4\pi} \int \frac{I\, dl \sin\phi}{r^2}$$

$\cos\theta = \frac{d}{r}$; $r = d \sec\theta \Rightarrow l = d\tan\theta$

$dl = d\sec^2\theta\, d\theta$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\theta \cdot d\sec^2\theta \cdot \sin\phi}{d^2 \sec^2\theta} = \frac{\mu_0 I}{4\pi d} \int_{-\pi/2}^{\pi/2} d\theta \cdot \cos\theta = \frac{\mu_0 I}{2\pi d}$$

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2. Maxwell's Equations:

a. $\vec{\nabla} \cdot \vec{B} = 0$ $\Updownarrow$ $\oint_S \vec{B} \cdot d\vec{a} = 0$ "No-name Law"

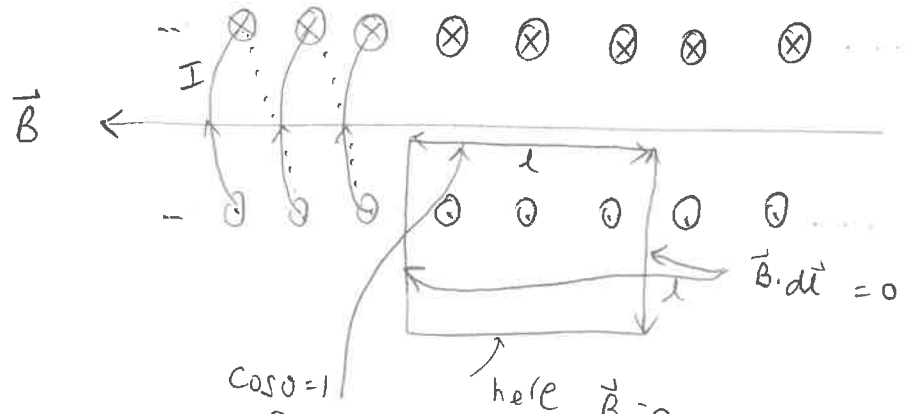
b. $\vec{\nabla} \times \vec{E} = -\partial_t \vec{B}$ $\Updownarrow$ $\oint_L \vec{E} \cdot d\vec{l} = -\partial_t \left(\int_S \vec{B} \cdot d\vec{a} \right)$ "Faraday's Law"

c. $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ $\Updownarrow$ $\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I$ "Ampere's Law"

3. Ampere's Law: $\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I_{\underline{encl}}$

Note $\underline{\text{Enclosed}}$ in I on R.H.S.

Task: Infinite solenoid ?



$$\int \vec{B} \cdot d\vec{l} = B \cdot l = \mu_0 I_{encl} = \mu_0 \underline{\underline{N}} I$$

N: Total winding in 'l'

n: windings per unit length

$$B = \frac{\mu_0 N I}{l} = \frac{\mu_0 \, n \cdot l \, I}{l} = \mu_0 n I$$

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4. Faraday's Law $\Longleftrightarrow$ Lenz's Law

Notice: $\vec{\nabla} \times \vec{E} = -\partial_t \vec{B} \Rightarrow \oint_L \vec{E} \cdot d\vec{l} = -\partial_t \underbrace{\int_S \vec{B} \cdot d\vec{a}}_{\text{flux} - \Phi}$

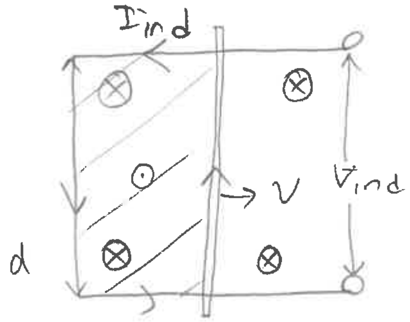
force · distance = unit work = "potential"

$$\Rightarrow \qquad V = -\partial_t \Phi_B$$

Notice $\Phi = \int_S \vec{B} \cdot d\vec{a} = B \cdot A$

area enclosed by induced current.

Task: #2



From Lenz's Law = Induced $\vec{B}(I_{ind})$ should oppose applied $\vec{B}$ $\Rightarrow$ gives I_{ind}

Now Calc. $\Phi_B = ?$ $\underbrace{\int \vec{B} \cdot d\vec{a}}_{S = \text{enclosed by } I_{ind}}$ $\underline{\cos\theta = 1}$ $B \cdot \boxed{A} = B \cdot d(vt)$

by Faraday's Law: $V_{ind} = -\partial_t \Phi_B = -\frac{\partial}{\partial t}(B \cdot dvt) =$

$$= -Bdv$$

Note $V_{ind} = -\partial_t \Phi_B$

This negative sign is Lenz's Law

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5. Magnetic field, Vector potential

Electrostatics	Magnetostatics
in free space:	
$\vec{\nabla} \cdot \vec{E} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
$\nabla \cdot (\nabla V) = 0 = \nabla^2 V$	$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

Biot - Savart's Law for $\vec{A}$

$$\vec{A}(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times d\vec{l}}{r}$$

no - squared Law

in magnetostatics: $\vec{\nabla} \times \vec{A} = \vec{B}$

only when $\partial_t \vec{E} = 0$, otherwise beware

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6. Inductance : L (Self)

$\Phi_B = L \cdot I_{ind}$; by faraday's Law: $V_{ind} = \partial_t \Phi$

inductance

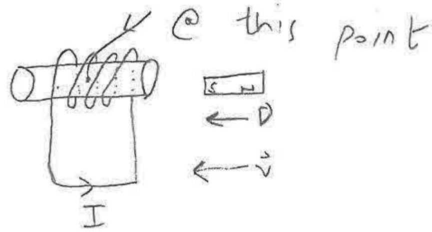
$$V_{ind} = \partial_t (L \cdot I_{ind}) = L \left(\frac{\partial I}{\partial t} \right)$$

← use this in Kirchoff's Rules

1. Lenz Law: "Emf induced such that it opposes the flux change"

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

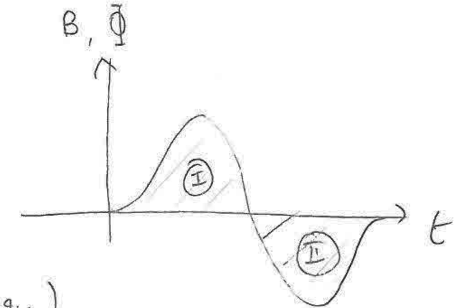
a.



Tasks:

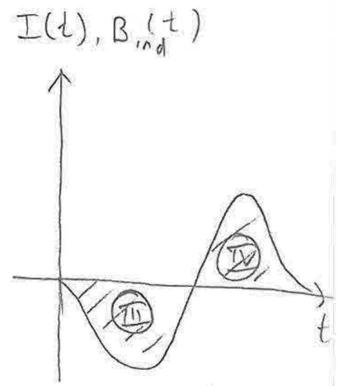
(i) plot $\vec{B}_z(t)$?

same as $\Phi(t)$



Note: I = II (Cons. of Energy)

(ii) Now plot the $I(t) \longrightarrow B_{ind}(t)$?



$$\left.\begin{aligned} III &\rightarrow -I \\ IV &\rightarrow -II \end{aligned}\right\}$$ This is Lenz Law statement.

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2. Faraday's Law:

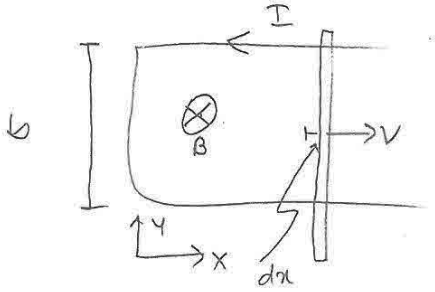
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \iff \oint_L E \cdot dl = -\int_S \frac{\partial B}{\partial t} \cdot d\vec{a}$$

$$V_{emf} = -\frac{\partial}{\partial t} \int_S B \cdot d\vec{a}$$

$$= -\frac{\partial \Phi_B}{\partial t}$$

a. $\mathcal{E}_{emf} = -\frac{d\Phi_B}{dt} = \frac{\int \vec{B}\cdot d\vec{a}}{dt} : V$ This has direction

Task: (i) Is $\int B \cdot d\vec{a}$ changing with time

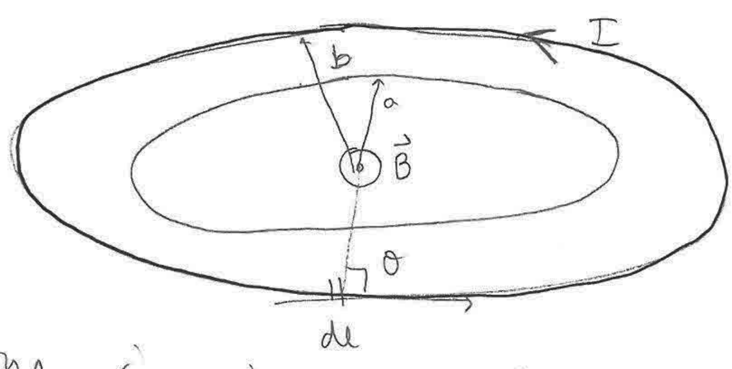


Notice $|d\vec{a}| = b\,dx$

(ii) Indicate direction of I ?

(iii) Calc. $\mathcal{E}mf = -\frac{d\Phi}{dt}$? $= -Bbv$

3.



Assume the ring is conductor

Task:

(i) which direction would 'I' go when $\vec{B}$ is turned off ?

(ii) $\mathcal{E}mf = ? = \pi a^2 \cdot \frac{dB}{dt}$ $(= \oint_C \vec{E}\cdot d\vec{l}$

(iii) $\vec{E}$? indicate direction : (Always along current)

$$d\tau = \vec{r} \times d\vec{F}$$

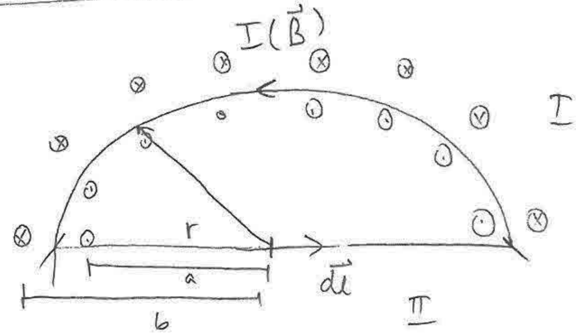
$$= \vec{r} \times dq \cdot \vec{E} = b \, . \, \lambda d\vec{l} \, . \, \vec{E} \, . \, \sin\theta$$

$$\tau = \int d\tau = \int b . \lambda d\vec{l} \, . \, \vec{E} \, . \sin\theta$$

(iii) what is $\sin\theta$? $(= \pi/2)$

(v) $\tau = \int b\lambda \; \vec{E} \cdot d\vec{l} = b\lambda \int \vec{E} \cdot d\vec{l} = ?$

4.



Ampere's Law: $\int \vec{B} . d\vec{l} = \mu_0 \vec{J} = \mu_0 (N . I)$

a. Task:

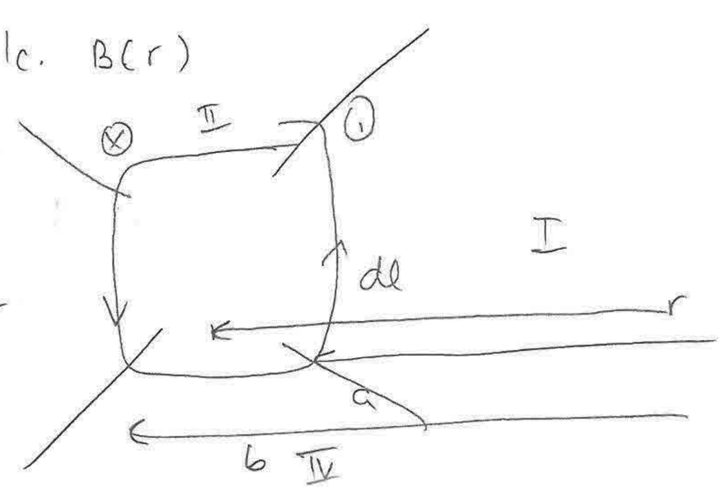
(i) Indicate direction of $\vec{B}$ in I , II ?

(ii) $\oint_C \vec{B} \cdot d\vec{l} = \int B_I \cdot dl_I + \int B_{II} \cdot dl_{II} = ?$

$\mu_0 N . I = B . 2\pi r$ 0

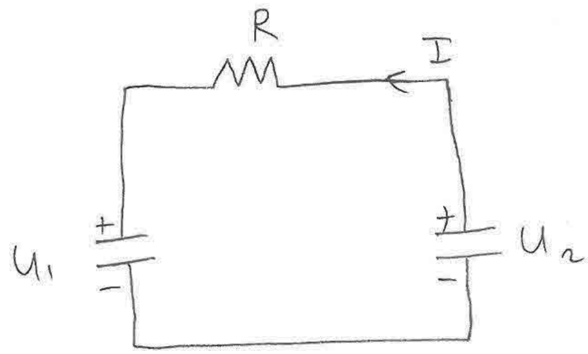
Calc. $B(r)$

b.



B. $\Phi \cdot ? = L \cdot I \; ? \propto I$

5.



a. Task.

(i) write Kirchoff's Loop Rule for $\uparrow$

$$\left(U_2 - I(t)R - U_1 = 0 \right)$$

Note: @ $t=0 \Rightarrow U_2(0) = \frac{Q_2^0}{C_2}$; $U_1(0) = \frac{Q_1^0}{C_1}$

& $I = -\dot{Q}_1 = \dot{Q}_2$

@ $t = \infty \Rightarrow I(t) = 0$

(ii) write Everything in terms of Q_i

$$\partial \left(\frac{Q_2}{C_2} - I(t)R - \frac{Q_1}{C_1} \right) = 0$$

$$\frac{I}{C_2} - \dot{I}R - \frac{I}{C_1} = 0 \Rightarrow \text{Solve } I.$$

b. Note: $V_L = L \frac{dI}{dt}$

just replace $I(t)R \rightarrow L \frac{dI}{dt} = L\dot{I}$

Solve for I.