# HW 1 - Week 1



Micro lecture:

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Differential Equations: Recap

1. I order homogeneous
2. I order inhomogeneous
3. II order homogeneous
4. II order Inhomogeneous

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1. I order homogeneous separate variables

$$\frac{dy}{dt} = -p(t)\,y$$

$$\int \frac{dy}{y} = \int -p(t)\,dt$$

$$\ln y = R(t) + C$$

$$y = e^{R(t)+C} = e^{C}\boxed{e^{R(t)}}$$

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2 I order inhomogeneous: Use ansatz: $f(x) = \underline{z}e^{\lambda x}$

$$a\frac{df(x)}{dx} + b\,f(x) + C(x) = 0$$

$$a z^{2} e^{\lambda x} + b z^{2} e^{\lambda x} + C(x) = 0$$

* Plug in $ze^{\lambda x}$

& solve $a f'(x) + b f(x) = 0 \rightarrow f_0(x) = ?$

* solve again using one of the functions below. $\rightarrow$ $f_p(x) = ?$

| $C(x)$ | $f_p(x)$ |
|---|---|
| $n^{th}$ order poly. | $A_n x^n + A_{n-1} x^{n-1} + \dots + A_0 x^0$ |
| $be^{kx}$ | $Be^{kx}$ |
| $be^{kx}$($n^{th}$ order poly) | $e^{kx}(A_n x^n + A_{n-1} x^{n-1} + \dots + A_0)$ |
| $be^{kx} \sin(ax)$ | $e^{kx}(B\sin(ax) + C\cos(ax))$ |
| $b\sin(ax)$($n^{th}$ order poly) & $b\cos(ax)$[$n^{th}$ order poly] | $(B\sin(ax) + C\cos(ax))[A_n x^n + A_{n-1} x^{n-1} + \dots + A_0]$ |
| Sum of any of above function | Sum of corresponding $f_p(x)$ |

$f(x) = f_0(x) + f_p(x)$

Substitute & check

3. II order homogeneous D. Eq

$$y''(t) + p(t)\,y'(t) + q(t)\,y(t) = 0$$

try $y(t) = e^{rt}$; substitute & solve for 'r'

$\Rightarrow$ 2 values of 'r' from quadratic Eq.

Combine them with $C_1 e^{r_1 t} + C_2 e^{r_2 t}$

If $r_1$ & $r_2$ are imaginary remember to use $y_0(t) = A\cos(\omega t) + B\sin(\omega t)$

4. II order inhomogeneous D. Eq

$$y''(t) + p(t)\,y'(t) + q(t)\,y(t) = s(t)$$

$\Rightarrow$ use same technique as for I order

* solve homogeneous first : $\rightarrow y_0(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

$y''(t) + p(t)y'(t) + q(t)y(t) = 0$

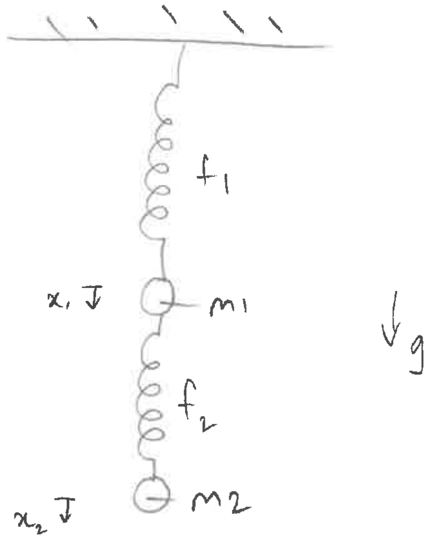
$\Downarrow$

* then use table & get $y_p(t)$

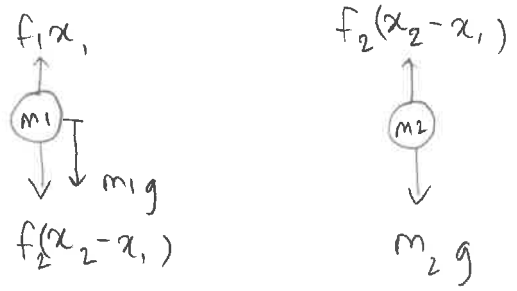
* $y(t) = y_0(t) + y_p(t)$, substitute back & solve again.

# Hw 1

1.

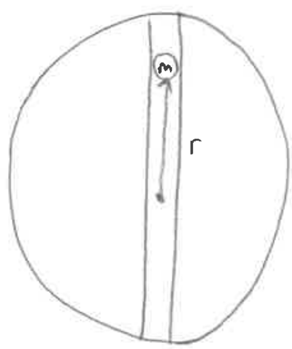



a. Force balance (




b. Use newton's Laws: $\sum_i F_i = ma = m\ddot{x}$

2.




a. find 'g' @ r $\longrightarrow$ use $F = \frac{G M_\oplus m}{r^2}$

note $M_\oplus$ changes with r.

apply Newton's II Law again

* only 1 force acting

b. Use some math hand book, but you should know solution of this family of D.Eq.

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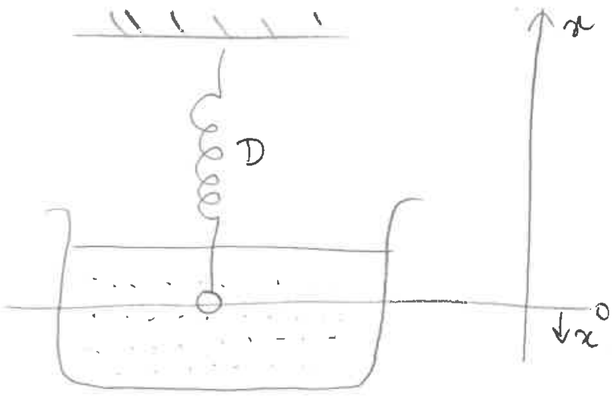
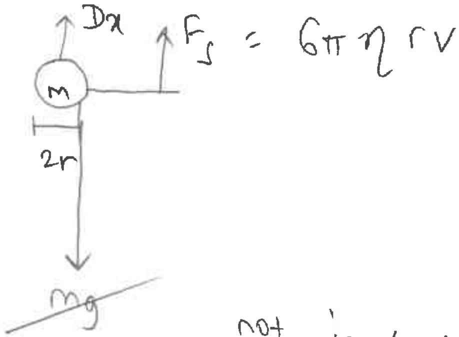
3




a. Note STOKES LAW: $F_s = -6\pi\eta r v = -6\pi\eta r\dot{x}$

force balance:

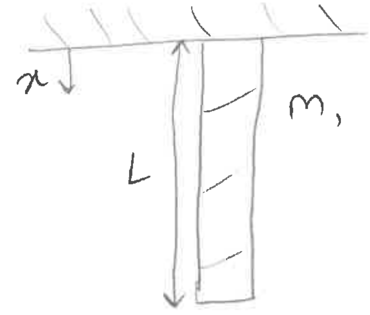



not included

b You should recognize family of D. Eq.

c. In weekly damped case, follow given math & solution will simplify.

4.



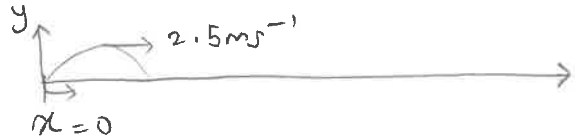

$\Rightarrow \mu = \frac{m}{L}$

Notice Tension, $F(x)$ is dependent on @ which point on rope you look at

$$F(x) = m(x)g = \frac{L-x}{L} g$$

then use $v = \sqrt{\frac{F}{\mu}}$

5.




a. Given:

$v = 2.5 ms^{-1}$
$f = 50 Hz$ $\Big\} \rightarrow$ Calc $v = \lambda f$.
$A = 2 cm$

$f = \frac{1}{T} \Rightarrow T = ?$ time period

wave: $y(x,t) = A \sin(\underset{2\pi f}{\omega} t - \underset{2\pi/\lambda}{k} x)$

b. Start wave @ $x = 3.75$ cm, instead of

@ $x = 0$ like in a.